Mathematics's uses in Different Engineering Fields

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ABSTRACT

Every engineering field makes extensive use of mathematics, especially applied mathematics. Many instances of the use of mathematics in mechanical, chemical, and electrical engineering are discussed in this study. The applications shown here are actual engineering applications, which may not be the same as those covered in many math textbooks. This essay's goal is to connect engineering with mathematics. Engineering challenges that need a lot of mathematics are challenging for many engineering students to tackle. The students have studied mathematics in the past (calculus, linear algebra, numerical analysis), but they frequently struggle to connect mathematics to engineering courses that require it. Engineering students may be inspired to better grasp their engineering difficulties by the examples provided, it is hoped. Also it is expected that mathematics lecturers can be encouraged to provide mathematics problems which are more related to engineering fields.

Keywords: mechanical, chemical, and electrical uses.

1. Introduction

Mathematics is the background of every engineering fields. Together with physics, mathematics has helped engineering develop. Without it, engineering cannot evolved so fast we can see today. Without mathematics, engineering cannot become so fascinating as it is now. Linear algebra, calculus, statistics, differential equations and numerical analysis are taught as they are important to understand many engineering subjects such as fluid mechanics, heat transfer, electric circuits and mechanics of materials to name a few. However, there are many complaints from the students who find it difficult to relate mathematics to engineering. After studying differential equations, they are expected to be able to apply them to solve problems in heat transfer, for example. However, the truth is different. For many students, applying mathematics to engineering problems seems to be very difficult. Many examples of engineering applications provided in mathematics textbooks are often too simple and have assumptions that are not realistics. See [8] for a good textbook which discusses mathematical modelling with real life applications. A lot of problems solved using Maple and MATLAB are given in [11]. The purpose of this paper is to show some applications of mathematics to various engineering fields. The applications discussed do not need advanced mathematics so they can be understood easily. The problems in the this paper have been solved using Maple, a symbolic programming language. For general introduction to Maple, see [12-13], for example. The first problem is about beam deflection (mechanical engineering), the second on the equation of state for refrigerants (chemical engineering) and the last on the illumination problem (electrical engineering).

2. Beam deflection for a cantilever with variable cross section

A cantilever with variable cross section is shown in Fig. 1. The beam carries a concentrated load P at its free end. This is a kind of problem for mechanics of materials for mechanical, civil and aeronautical engineering. The load given to the beam causes it to deflect. We will find the curve of deflection along the beam and determine the maximum deflection that occurs. The relationships between the bending moment M_x and the deflection Y is given by

$$M_x = EIY'' \quad (1)$$

Eq. (1) is given in most textbooks of strength of materials or structural engineering; see [1−3] for example. E is Young modulus of elasticity and I is moment of inertia of the beam. Collectively, EI is called the flexeural ridity of the beam. In Eq. (1), the downward deflection is taken to be positive; it can be negative as adopted in [1]. The moment of inertia is positive if the direction clockwise. Eq. (1) can be written as

$$y'' = \frac{M_x}{EI} \quad (2)$$

Eq. (2) must be integrated twice in order to give the deflection at any point along the beam.

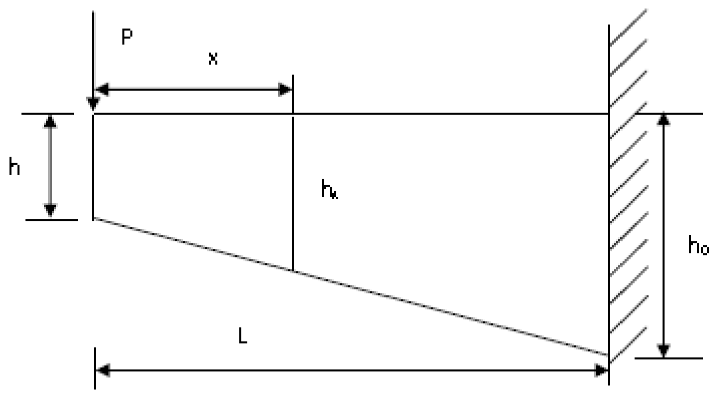

Fig. 1: A cantilever with variable cros section

For a concentrated load, the bending moment is given by

$$M_x = -Px \qquad (3)$$

where x is the distance from the left. For a beam with a rectangular cross section, the moment of inertia is given by [1]

$$I_x = \frac{1}{12} b h^3$$

where

$$h_x = h + (h_0 - h)\frac{x}{L}$$

So,

$$I_x = \frac{1}{12} b \left[h + (h_0 - h)\frac{x}{L}\right]^3 = \frac{1}{12} b \left[h + h_0\left(1 - \frac{h}{h_0}\right)\frac{x}{L}\right]^3$$

which can be written as

$$I_x = \frac{1}{12} b h_0^3 [m + (1-m)p]^3 = I_0 [m + (1-m)p]^3$$

where

$$I_0 = \frac{1}{12} b h_0^3$$

$$m = \frac{h}{h_0}, \quad 0 < m \leq 1 \quad \text{and } p = \frac{x}{L}, \quad 0 \leq p \leq 1$$

So Eq. (2) becomes

$$y'' = -\frac{Px}{EI_0[m + (1-m)p]^3} = -\frac{PpL}{EI_0[m + (1-m)p]^3} \qquad (4)$$

Eq. (4) can be written dimensionless by dividing it by PL/EI_0. We then have

$$y''\times(EI_0/PL) = Y(p) = -\frac{p}{[m + (1-m)p]^3} \qquad (5)$$

Integrating Eq. (5) twice gives

$$Y(p) = \frac{1}{(1-m)^2}\left\{-\frac{1}{2}\frac{m}{[m + (1-m)p]^2} + \frac{1}{[m + (1-m)p]}\right\} + C_1 \qquad (6)$$

$$Y(p) = \frac{1}{(1-m)^3}\left\{\frac{1}{2}\frac{m}{[m + (1-m)p]} + \ln[m + (1-m)p]\right\} + C_1 p + C_2 \qquad (7)$$

In Eq. (6), Y'(p) is the slope of the curve of the deflection. C_1 and C_2 are constants of integration which can be found as follow. At p =1, Y'(1) = 0 and Y(1) = 0. Without derivation, the values of C_1 and C_2 are given as

$$C_1 = \frac{1}{2}\frac{m-2}{(1-m)^2}; \qquad C_2 = \frac{1}{2}\frac{m^2 - 4m + 2}{(1-m)^3} \qquad (8)$$

C_1 and C_2 are substituted back to Eqs. (6) and (7).

We are interested to find the slope and the deflection at the free end. At the free end (p = 0), we then have

$$Y'(0) = \frac{1}{2m} \qquad (9)$$

$$Y(0) = \frac{1}{2}\frac{3 + 2\ln(m) - 4m + m^2}{(1-m)^3} \qquad (10)$$

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The true slope and deflection are found by multiplying Eqs. (9) and (10) by PL/EI_0. So,

$$y'(0) = \frac{1}{2m}\frac{PL}{EI_0} \tag{11}$$

$$y(0) = \frac{1}{2}\frac{3 + 2\ln(m) - 4m + m^2}{(1 - m)^2}\frac{PL}{EI_0} \tag{12}$$

For a rectangular beam, $h_0 = h$ or $m = 1$ so we have

$$y'(0) = \frac{PL}{2EI_0} \tag{13}$$

$$y(0) = \lim_{m \to 1}\frac{1}{2}\frac{3 + 2\ln(m) - 4m + m^2}{(1 - m)^2}\frac{PL}{EI_0} = -\frac{PL}{3EI_0} \tag{14}$$

3. First order irreversible series reactions

There are a lot of applications of mathematics in chemical engineering; see for example [5−7]. Examples of applications solved by Maple are given in [5]. Various applications in chemical engineering such as kinetic modeling, diffusion/reaction problems, computational fluid dynamics, chemical reaction engineering and control problems are given in [6]. A case study discusing the dsign of the biotreatment systems is discussed in [7]. Thermodynamic properties and equations of states are discussed in various books; see [9-10], for example. The folowing example is adapted from [5]. Consider the first order irreversible series reactions

$$\left(A \xrightarrow{k_1} B \xrightarrow{k_2} C\right)$$

The governing equations for this reaction scheme are given as

$$\frac{dC_A}{dt} = -k_1 C_A \tag{15}$$

$$\frac{dC_B}{dt} = k_1 C_A - k_2 C_B \tag{16}$$

where k_1 and k_2 are rate constants and the initial conditions are $C_A(0) = 1$ mol/l, $C_B(0) = 0$, and $C_C(0) = 0$. The concentration of species C (t) at any time is given by the material balance:

$$C_C = 1 - C_A - C_B \tag{17}$$

Using Maple, the solution of Eqs. (15) and (16) is given by

$$C_A = e^{-k_1 t} \tag{18}$$

$$C_B = \frac{k_1\left(e^{-k_1 t} - e^{-k_2 t}\right)}{k_2 - k_1} \tag{19}$$

For illustrations, we will take $k_1 = 2$ and $k_2 = 3$. So, we then have

$$C_A = e^{-2t} \tag{20}$$

$$C_B = 2e^{-2t} - 2e^{-3t} \tag{21}$$

$$C_C = 1 - 3e^{-2t} + 2e^{-3t} \tag{22}$$

As t approaches infinity, both C_A and C_B will approaches zero and C_C approaches 1. However, in practice, at t = 5, both C_A and C_B are quite small and they can be taken to be zero while C_C is very near to 1. Fig. 2 shows the graph of the three concentrations.

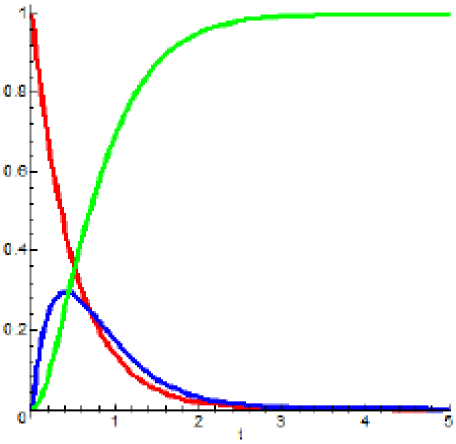

Fig. 2: Concentrations of C_A (red), C_B (blue) and C_C (green)

We will now explore the reaction equations. First, we want to know when C_A and C_B have the same concentration. By equating Eq. (20) to Eq. (21) we have

$$e^{-k_1 t} = \frac{k_1(e^{-k_1 t} - e^{-k_2 t})}{k_2 - k_1}$$

which can be solved to give

$$t^* = \frac{\ln\left[\frac{k_1}{2k_1 - k_2}\right]}{k_2 - k_1} \quad (23)$$

Substituting t* to Eq. (18) or (19) produces

$$C_A{}^* = C_B{}^* = \left[\frac{k_1}{2k_1 - k_2}\right]^{\frac{k_1}{2k_1 - k_2}} \quad (24)$$

With $k_1 = 2$ and $k_2 = 3$ we then have $t^* = \ln(2)$ and $C_A{}^* = C_B{}^* = ¼$.

Now, the next question: what happens if $k_1 = k_2$? If we substitute $k_1 = k_2$ to Eq. (19) we will get the division of 0/0, which is indeterminate. However, we can check the limit of C_B as $k_2 \rightarrow k_1$. Here, we have

$$C_B = \lim_{k_2 \rightarrow k_1} \frac{k_1(e^{-k_1 t} - e^{-k_2 t})}{k_2 - k_1} = k_1 t e^{-k_1 t} \quad (25)$$

So, for $k_1 = 2$ we have

$$C_A = e^{-2t}, \quad C_B = 2te^{-2t}, \quad C_C = 1 - e^{-2t} - 2te^{-2t}$$

What is the value of t which will make C_B maximum? Differentiating Eq. (19) with respect to t and equating it to zero gives

$$\frac{dC_B}{dt} = \frac{k_1\left(k_1 e^{-k_1 t} - k_2 e^{-k_2 t}\right)}{k_1 - k_2} = 0$$

which can be solved to give

$$t_{max} = \frac{\ln\left[\frac{k_1}{k_2}\right]}{k_1 - k_2} \tag{26}$$

Substituting t_{max} to Eq. (19(gives

$$C_{B,max} = \frac{k_1}{k_1 - k_2}\left[\left(\frac{k_1}{k_2}\right)^{-k_2/(k_1-k_2)} - \left(\frac{k_1}{k_2}\right)^{-k_1/(k_1-k_2)}\right] \tag{27}$$

For $k_1 = 2$ and $k_2 = 3$, $t_{max} = 0.4055$ and $C_{B,\ max} = 0.2963$.

If $k_1 = k_2$, we can find find the limit of $C_{B,\ max}$ = as $k_2 \rightarrow k_1$. However, it is much easier if we just differentiate Eq. (25) with respet to t and equate the value to zero. We have,

$$\frac{dC_B}{dt} = k_1 e^{-k_1 t}(1 - k_1 t) = 0$$

which can be solved to give $t = 1/k_1$. Susbtituting to Eq. (25) gives $C_{B,\ max} = 1/e$. It can be seen that when $k_1 = k_2$, maximum concentration of CB is independent of the rate constants and the time to achieve it is inversely proportional to the rate constant k_1 [5].

First order reversible series reactions are a bit more complicated. For three concentrations, there are three linear, first order differential equations to solve. However, Maple can solve the problem easily; see [5] for the solution.

4. Illumination problems

This illumination problem is adapated from [11]. A courtyard is illuminated by two lights, where P_i is the illumination power and h_i the height of a lamp; see Fig. 3. The coordinates of the lamps are (0, h_l) and (s, h_2) where s is the horizontal distance between the two light sources. Let X = (x,0) be a point on the courtyard somewhere between the two lights. We wish to find a point X which will get the minimum illumination from the two lamps. In this problem, P_i is kW while h_i and s are in meter. We will use Maple to solve the problem.

From Fig. 3 we have

$$r_1^2 = h_1^2 + x^2\ , \quad r_2^2 = h_2^2 + (s - x)^2$$

Following [11], the light intensities from the two lamps at X are given by

$$I_1(x) = \frac{P_1}{r_1^2} = \frac{P_1}{h_1^2 + x^2}\ , \qquad I_2(x) = \frac{P_2}{r_2^2} = \frac{P_2}{h_2^2 + (s - x)^2}$$

The illumination $Il_i(x)$ at point x from each lamp is given by $Il_i(x) = I_i(x)\sin(\alpha_i)$ so we then have

$$Il_1(x) = \frac{P_1 h_1}{\sqrt{(h_1^2 + x^2)^3}}, \qquad Il_2(x) = \frac{P_2 h_2}{\sqrt{(h_2^2 + (s - x)^2)^3}} \tag{28}$$

For easy typing, we will modify the symbols used by writing P_1 = p, P_2 = Q, h_1 = a and h_2 = b. The total illumination at the point is then given by

$$C(x) = I_1(x) + I_2(x) = \frac{pa}{\sqrt{(a^2 + x^2)^3}} + \frac{qb}{\sqrt{(b^2 + (s - x)^2)^3}} \quad (29)$$

C(x) will be minimum when dC(x)/dx = 0. So,

$$\frac{dC(x)}{dx} = -\frac{3pax}{(a^2 + x^2)^{5/2}} + \frac{3qb(s - x)}{(b^2 + (s - x)^2)^{5/2}} = 0 \quad (30)$$

However, Maple cannot solve this general problem. To help it, we remove the radical by squaring the expression. We have

$$\frac{p^2a^2x^2}{(a^2 + x^2)^5} = \frac{q^2ab^2(s - x)^2}{(b^2 + (s - x)^2)^5} \quad (31)$$

Expanding this will produce a degree-12 polynomial. Once again, Maple cannot solve it. For an illustration, take p = 1 kW, q = 2 kW, a = 4 m, b = 5 m and s = 10 m. Substituting the known values to Eq. (31) produces after expanding and arranging the result:

$$F(x) \equiv 21x^{12} - 100x^{11} - \ldots - 52428000x + 2621440000 = 0 \quad (32)$$

The complete expression is too long to be shown here!

Numerical solutions of Eq. (32) produces three real roots between $0 \le x \le 10$; they are 0.1541, 4.4112 and 9.9111. The value that makes C(x) minimum can be found by testing the first derivative of F(x) with x. If F'(x) > 0, C(x) will be minimum. Here, x = 4.4112 (or 441 cm from the left lamp). Substituting this value to Eq. (29) gives the minimum illumination to be 0.0427.

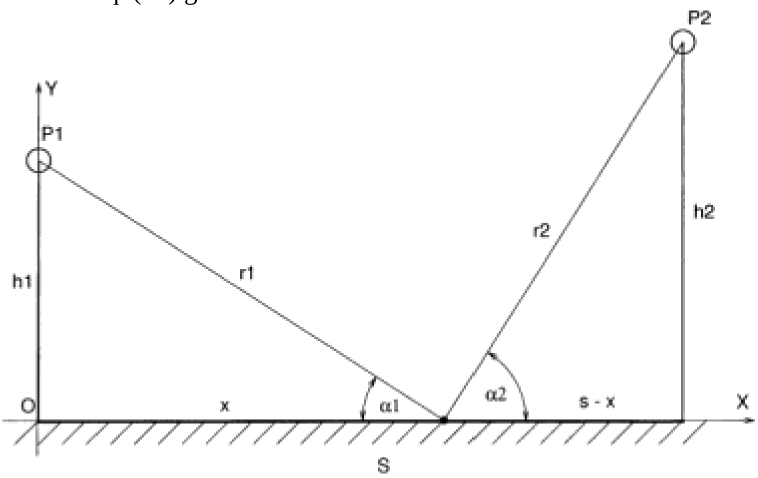

Fig. 3: Illumination problem

We can also solve Eq. (30) directly without trying to write it in the general form. Substituting p = 1, q = 2, a = 4, b = 5 and s = 10 to Eq. (30) yields

$$-\frac{12x}{(16 + x^2)^{5/2}} + \frac{30(10 - x)}{(25 + (10 - x)^2)^{5/2}} = 0$$

The solution of this non linear solution is 0.1541, 4.4112 and 9.9111. The result is the same as the previous one.

4. Conclusions

In this paper, three of applications of mathematics three different engineering fields have been presented. The problems are from real life. Each problem is solved using Maple. The problems do not need advanced mathematics to solve. Engineering students with strong background in calculus and numerical analysis can solve them without any difficulty. It is expected that the problems presented in this paper can motivate engineering students to understand mathematics better. Mathematics should be enjoyable as it has helped engineering evolved.

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