International Journal of Engineering, Management, Humanities and Social Sciences Paradigms (Volume 30, Special Issue of Nov 2018)
ISSN (Online): 2347-601X and Website: www.ijemhs.com

Some Studies and Innovations in Probabilistic Education

1.Ms. Swagatika Mohanty 2.Mr.Ganeswar Mahanta
Nalanda Institute of Technology, Bhubaneswar
Dept. of Basic Science & Humanities
E-mail ID: swagatikamohanty@thenalanda.com
ganeswarmahanta@thenalanda.com

ABSTRACT

Many suggestions for probability education were offered in the topic study group on probability at ICME 11. In order to integrate advancements, this study offers a succinct assessment of the major lines of inquiry into probability education that have been conducted around the world. Furthermore offered are additional areas for study and inquiry.

INTRODUCTION

Whether probability education needs to be seen as discrete and separate from statistics has been an ongoing debate for many decades. Nowadays, statistics seems to be dominant in school education and data handling has been a key theme as part of the movement of mathematics for all. Conversely probability is thought to be harder and less relevant. Nevertheless, probability is an important discipline in its own right, and does contain the key underpinning concepts to understand and use data sensibly.

This paper focuses on the international research on probability issues in education, mainly derived from ICME 11 (2008) in Mexico. This study group, also linked to IASE included all the major themes being studied internationally in probability education. A thorough system of peer review, using an international panel of experts, ensured that the papers at ICME 11 were both well written and also covered the key areas of research being undertaken across the world. The full papers from the conference are available on the ICME 7 site in the links below. Most of the papers have been developed further, including many interactive features (such as links to related and background research ideas) and can be found in Borovcnik & Kapadia (2009), which can also be accessed electronically. These themes of international research provide a valuable bridge between international research and themes in Britain on the teaching of probability, which is often subsumed under data handling or statistics.

Probability education has not been a central focus of the research community in the last three decades since the theoretical framework espoused by Kapadia & Borovcnik (1991) appeared. Jones (2005) on "Exploring probability in Schools" has largely followed the new paradigm in educational research, which is empirically oriented. Designs of teaching sequels are administered to students and analysed. Sometimes, beliefs and attitudes of teachers are empirically investigated. Only one of the contributions in Jones is philosophically oriented: Batanero, Henry, & Parzysz (2005) give a summary of the philosophical debate on the interpretations of probability and discuss its implications on teaching.

Analysis of the subject matter is still dominated by Heitele's fundamental ideas (1975), which seem to be more a description of the main chapters of a probability textbook than an analysis of the concepts from a more general perspective and their purpose. The educational debate is being revived by the more recent endeavours to explain the concepts of risk (see Gigerenzer 2002), which comes from societal needs taken up by cognitive psychologists and thereby attracts more attention in the community of educationalists. However, the fundamental ideas of probability, in line with those discussed for statistics by Wild & Pfannkuch (1999) are still awaiting elaboration. Some starting points may be found in Pratt (2005) though the CERME working group, too, is mainly devoted to the empirical paradigm.

The authors support the ideas and results of Fischbein (1987) who elaborates on intuitions and their impact on understanding (and accepting) probabilistic concepts. Raw primary intuitions of individuals are revised by teaching interventions and changed to secondary intuitions, which should and could help to handle with the formal sides of the concepts as a companion. Such ideas have the potential to describe any learning process; research begins with the learning process of an individual or a group on the research topic of interest and later becomes a learning process within the wider community when research papers are published, Fischbein's ideas might also orientate the way a research community exchanges its results and enriches the discussion and makes progress.

Despite the fact that a multitude of new technologies is available now in the era of information technology, and multi-media is spreading to all corners of life, publication in research has hardly changed. Textbooks for study are changing gradually, a few hypertexts make use of the possibilities of new media but for research publications it seems that times have not changed yet. Borovcnik (2007) has analysed the consequences of new technologies on applications and on educational endeavours; more endeavour is needed from the research community to improve its communication. Kapadia & Borovcnik (2009) have taken up the challenge of innovative publication (including multi-media and more) in the age of web 2.0. In what follows, we will focus on the content, the main streams of research in educational probability by classifying the international endeavours to obtain new insights on the teaching of probability.

Central themes in international probability education research

Probability and statistics have been part of school mathematics for less than 40 years and complement the traditional topics of arithmetic, algebra and geometry. Statistics is part of the curriculum in virtually all countries but ideas of probability may only be introduced for older pupils. Application-oriented statistics is undisputed in its relevance, but the place of probability is more ambivalent. Reduction of probability to the classical conception, mainly based on combinatorics, and its perception as a solely mathematical discipline with its connection to higher mathematics, are sometimes used as arguments to abandon it in favour of the statistics part. However, there are key reasons for a strong role for probability within mathematics curricula:

1. Misconceptions on probability affect people's decisions in important situations, such as medical tests, jury verdicts, investment, assessment, etc.
2. Probability is essential to understand any inferential procedure of statistics.
3. Probability offers a tool for modelling and "creating" reality, such as in physics.
4. The concepts of risk (not only in financial markets) and reliability are closely related to and dependent upon probability.
5. Probability is an interesting subject in its own right and worthy of study.

The challenge is to teach probability in order to enable students to understand and apply it, by creating approaches that are both accessible and motivating. Both, the frequentist and subjectivist views of probability, and connections of probability to practical applications should be taken into account. Simulation is one such strategy, as is visualisation of abstract concepts. The use of technology helps to reduce the technical calculations and focus the learner on the concepts instead. The world of personal attitudes and intuitions is another source for success or failure of teaching probability. The main themes in the research, which nevertheless do overlap are: Conditional Probability and Bayes' Theorem; The School Perspective: Pre- and Misconceptions; The Teachers' Perspective: Pre- and in-service Courses; Impact of Technology; Fundamental Ideas.

Conditional Probability and Bayes' Theorem

Conditional probability and Bayesian inference are important ingredients of university teaching, including courses for non-mathematical students. Many different types of errors have been investigated in isolation. According to C. Batanero and C. Diaz (Spain), there is neither a study investigating connections between various types of misconceptions, nor an analysis whether misconceptions are related to mathematical knowledge, i.e. whether they decrease with better achievement in mathematics. Consequently, they have developed a test with (mainly familiar) items, and administered it to university students. Results were analysed with factor analysis. Though some phenomena remained even with higher mathematics education, there was a significant decrease in misconceptions with a higher level of mathematics. For interrelations between several misconceptions, the results were less optimistic as these misconceptions seemed to be quite isolated. As a consequence, endeavour in probability education has to be fostered and misconceptions still have to be continually and repeatedly stressed in teaching in order to facilitate students' understanding.

P. Huerta (Spain) identifies a serious flaw of some existing research which does not take the structure of the posed problems into account. He has classified the mathematical structure of "ternary problems" into 20 different types of problems with conditional probabilities of which only one subclass (and from it mainly one type of task) has been used in existing research. A graph with all problems has been developed to visualise the grade of difficulty of a special problem at hand. Applying this deep structural analysis, Huerta has developed a plan for future empirical research to cover all types of conditional probability problems to enhance the insights which might be gained.

The School Perspective: Pre- and Misconceptions

There has been a trend away from misconceptions, which may be changed by suitable teaching, towards pre-conceptions. Such a change of focus in research may be traced in current empirical research.

D. Abrahamson (USA) designed an experiment with a single child (Li, 11 years), using in an in-depth interview after a teaching phase in a classroom environment where an urn experiment was replaced step by step by the computer environment. Abrahamson analysed the learning trajectory of the child and how the interaction of the representation of the notions by different media influenced learning (see below), starting with a 4-block scooper to pick out blue and green marbles, as illustrated below. As a valuable side effect of the approach, the histogram can be visually linked (with a "greenish" impression) to the proportion of green marbles in the urn.

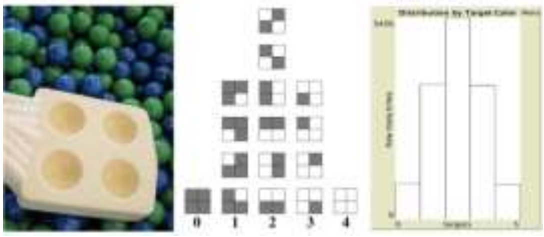

Figure 1. Abrahamson's "four block scooper" as a unifying element.

The empirical studies by F. Chiesi & C. Primi (Italy) and L. Zapata (Colombia/USA) dealt with heuristics in the tradition of Kahneman & Tversky, especially the changes in "negative" and "positive recency" with age. They compared 9, 11, and 25 years olds in order to imitate a longitudinal survey. Interestingly, their study showed an increase of the normative (correct) solution first (from age 9 to 11), which dropped again (age 25). They found that the bias towards "negative recency" decreased (from age 9 to 11) first and then increased again (age 25). However, the "positive recency" decreased (9 to 11) and was unchanged amongst adults (age 25). More in-depth investigations would clarify what is happening, and whether such a "development" can really be confirmed (of course it is not truly longitudinal).

Conditional probabilities are considered to be difficult; Bayesian inference is no less difficult. L. Martignon and S. Krauss (Germany) have worked with 10 year olds on these ideas and their precursors. They started with Wason cards to initiate learning steps in the children.

Which cards do you necessarily have to turn around in order to check, whether the following rule holds for the set of 4 cards? "If one side of a card exhibits a vowel, its other side must exhibit an odd number"

E **7** **K** **2**

Figure 2. The original Wason cards.

Children had difficulties but improved with contexts closer to everyday life. The researchers moved on to use tinker cubes (multi-cubes) to build towers, so that both sides could be seen simultaneously and then promoted learning steps in proportional thinking, reporting encouraging results from their pilot projects.

K. Rolka and S. Prediger (Germany) studied 12 year olds playing a game of luck with tokens moved forward on a playing board by the result of various sorts of dice. The discussion amongst the children revealed how they interacted and argued fiercely in favour of preferred strategies, learning from their *common* struggle for an optimum strategy. The social situation of the class with the children interacting in their discussion featured strongly.

The Teachers' Perspective: Pre- and in-service Courses

There are pitfalls in the interpretation of results from statistical tests or from confidence intervals. These originate from the reduction of the interpretation of probability to situations, which may be repeated independently in the same manner. On this issue there has been a vigorous debate not only in the foundations of statistics but also in the didactical community. Ö. Vancsó (Hungary) has developed a parallel course in classical *and* Bayesian statistics. He believes that it is a false dichotomy, to teach *either* classical statistics *or* Bayesian statistics, as both offer a consistent theory of probability. He has tried and refined his ideas in several cycles in teacher pre-service education and reported his positive experiences: "Now I have really understood what is meant by confidence intervals" one of his students exclaims.

An interesting extra-curricular activity was explored by H. Trevethan (Mexico) who described a project in the context of a science fair. A pair of students worked on a project to present a game of chance. There were several advantages. These included the autonomous activity of the students, their own responsibility, presenting in public etc. The game was "Shut the box", which is certainly open to varied stochastic strategies as different dice can be chosen to play the game. This authentic (and not artificial) transfer of responsibility could well be taken up more often in teaching in class. Mathematically, conditional probabilities and Bayes' theorem are the key concepts to develop winning strategies.

K. Lyso (Denmark) used a battery of standard tasks covering the main primitive concepts including two-stage experiments. The distinctive feature was on the discussion about which solutions are feasible, or which reconstruction of the task made sense and therefore led to a sensible solution even if it did not coincide with the "normative" solution. One result was the documentation of an inclination to reformulate tasks. The students sometimes reformulated two-stage experiments into one-stage tasks getting a wrong answer but found it hard to see why their reconstruction was misleading:

L. Zapata (Columbia/USA) investigated well-known tasks from Kahneman & Tversky in order to clarify what may be learned from new as well as more experienced teachers. She has tried to derive meta-knowledge from her in-depth interviews with teachers. Surprisingly, or possibly unsurprisingly, novice teachers repeated the same misleading intuitive conceptions as their students and thus were not really able to help them. It may be that probability is much more prone to such difficulties than other topics in mathematics. This is confirmed by V. Kataoka (Brazil) who ran a series of workshops of in-service education. One special experiment used in the workshop illustrates the importance of suitable models and data sampled by randomness (when do you really have data from *random* samples?)

> We break a stick randomly into three pieces. Afterwards, the subjects are asked to form a triangle of the three pieces. Finally the success rate is determined with which triangles actually could be formed. Try it with spaghetti – without explaining in advance what you plan.

Success rates of 75% are not rare. In contrast to it, there are (at least) two models for randomly breaking the stick (with 25% and 19% success rates). The obvious discrepancy between the theory and the model lets us gradually start to doubt whether we can break the stick truly randomly into 3 parts. As a conclusion, relative frequencies might sometimes be of no value to estimate an unknown probability. This enriches the usual discussion about the convergence of relative frequencies by focussing on the underlying assumption of randomness of the data. Analogous examples are abundant but are less emotionally laden than spaghetti.

S. Anastasiadou (Greece) has developed a battery of simple items to research relations between algebraic and graphical skills in student teachers. She used similarity diagrams and came to the conclusion that there is a widespread lack of skill to change between different representations of a task or a notion. This lack of conceptual flexibility seemed to hinder a deeper comprehension of the notions. With different representations, students seem to learn *different* concepts – they do not necessarily notice that the representations deal with the same notion only in a different form.

Impact of Technology

Technology can be viewed in at least two very distinct ways. In one aspect is the media used such as Powerpoint or interactive use of computers by students. The other aspect relates to the software tools used. Some software is generic (eg Excel) and some software is designed specifically for probability such as Fathom. In practice there is more software relating to statistics, though probability software is growing. Fathom and Tinkerplot can be used for efficient calculations and for illustrating key ideas such as the concept of distribution and the law of large numbers, as noted by S. Inzunsa (Mexico) and R. Peard (Australia).

New media indirectly form the backbone of the research of D. Pratt, writing with R. Kapadia (England) on shaping the experience of naïve probabilists. Sequences of the programme ChanceMaker supplied new and challenging experiences to learners in order to shape their intuitions and strategies. There are new challenges for designers of software and teachers using this software. In a fusion of control over the initial parameters (via randomness) and representations of results (histograms for the distribution of data or statistics like the mean), new insights into randomness have been generated. A new world of up-to-date unknown intuitions might emerge, which would affect concepts and their understanding.

J. Watson and S. Ireland (Australia) reported the results of in-depth interviews covering issues on the relations between empirical and theoretical aspects of probability. The class of 12 year olds undertook coin tossing and tabulation of results, followed similar experiments with Tinkerplot, which is becoming a popular piece of software. This widened the children's experience. Some questions remained open for further scrutiny. Can the computer really generate randomness? How can one read diagrams from the software correctly? How can we ensure that the children have sufficient experience in proportional thinking?

Fundamental Ideas

The fundamental ideas of probability include random variables, distribution, expectation, and relative frequencies, as well as the central limit theorem. R. Peard (Australia) has gone to the roots of the subject with questions from games of chance. Games of chance have been partially discredited by their closeness to combinatorics (which is not always easy to understand) and by their artificiality (we want to teach real applications to our students). However, games of chance have spread widely, such as lotteries, and developed to become an important business sector, which is still growing.

M. Borovcnik (Austria) has studied some peculiarities of stochastic thinking, which make it so different from other approaches:

- There is no direct control of success with probabilities – the rarest event may occur and "destroy" the best strategy.

- Interference with causal re-interpretations may lead a person completely astray.
- Our criteria in uncertain situations may stem from "elsewhere" and may be laden with emotions – probability and divination have a common source in ancient Greece.

With these features of stochastic thinking in mind, paradoxes like the stabilising of relative frequencies, even though new events have full-fledged variability, may not seem special. One difficulty lies in a primitive attribution of an ontological character of probability to situations. Probability does not exist – it is only *one of many views* to reflect on phenomena of the real world.

Perspectives for the future

We end with some assertions requiring further research endeavour in probability education:

- People use their experience in order to judge probabilities incompletely and – even worse – in a haphazard manner.
- People have difficulties in judging very small and very high probabilities especially if these are connected to adverse consequences.
- People are inclined to attribute equal chances to the – given or seen – possibilities, especially if there are just two.
- People attribute probabilities and process these into new ones neglecting even the most basic rules (e. g. all probabilities sum to 1).

We believe that sharing and testing ideas across different countries will help promote deeper understanding. In particular, further empirical testing using shared instruments will yield deeper insights.

References and Brief Bibliography

Batanero, C., M. Henry, and B. Parzysz. 2005. The nature of chance and probability. In *Exploring probability in schools: Challenges for teaching and learning,* ed. G. A. Jones , 15–37. New York: Springer.

Borovcnik, M. 2007. New Technologies revolutionize the applications of statistics and its teaching. Invited Paper Session 38: How modern technologies have changed the curriculum in introductory courses, 56. *Session of the ISI Lisboa*. Retrieved from www.stat.auckland.ac.nz/~iase/publications/isi56/IPM38_Borovcnik.pdf July 10, 2009.

Borovcnik, M., and R. Kapadia. eds. 2009. Research and Developments in Probability Education, *International Electronic Journal of Mathematics Education* 4 (3). www.iejme.com

Borovcnik, M., and R. Peard. 1996. Probability. In *International Handbook of Mathematics Education,* ed. A. Bishop, et.al., 239–288. Dordrecht: Kluwer.

Fischbein, E. 1987. *Intuition in Science and Mathematics. An Educational Approach*. Dordrecht: Reidel.

Gigerenzer, G. 2002. *Reckoning with risk: learning to live with uncertainty.* London: Penguin Books.

Heitele, D. 1975. An epistemological view on fundamental stochastic ideas. In *Educational Studies in Mathematics* 6: 187–205.

Jones, G., ed. 2005. *Exploring probability in school: Challenges for teaching and learning*. New York: Springer.

Kapadia, R., and M. Borovcnik eds. 1991. *Chance Encounters: Probability in Education*. Dordrecht: Kluwer Academic Publishers. Online at Google books: books.google.com/books?id=doqeKAICpQ4C&pg=PP1&dq=kapadia+borovcnik&hl=de&cd=1#v=onepage&q=&f=false.

Pratt, D., ed. 2005. Working Group 5 on "Stochastic Thinking". In *European Research in Mathematics Education IV. Proceedings CERME 4*, ed. M. Bosch , 481–630. Barcelona: Fundemi IQS – Universitat Ramon Llull. CD ROM. Retrieved from ermeweb.free.fr/CERME4/CERME4_WG5.pdf, July 10, 2009.

Wild, C., and M. Pfannkuch . 1999. Statistical thinking in empirical enquiry (with discussion), *International Statistical Review* 67: 223–265.