Highest Common Factor and Lowest Common Multiple Using the SESA Sutra

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Abstract

1. The Vedic Mathematics Sutra The Remainders By the Final Digit is utilised in this paper to demonstrate how the Vedic Mathematics Sutras can be used to find the highest common factors and lowest common multiples of two or more numbers. Polynomial expressions can also use this.

2. INTRODUCTION

Finding highest common factors and lowest common multiples of numbers and algebraic expressions are fundamental processes in mathematics with many applications.

3. HIGHEST COMMON FACTOR, HCF

This is also known as the Greatest Common Divisor or Greatest Common Factor.

HCF of Two Numbers

It is easily demonstrated (see for example Tirthaji's proof on page 2 of his chapter 10) that the HCF of two numbers is a factor of a sum or difference of any multiples of the numbers.

Tirthaji writes:[1]

> Let P and Q be the two expressions; H their H.C.F. and A and B the quotients (after their division by the H.C.F.)
>
> $\therefore \frac{P}{H} = A$; and $\frac{Q}{H} = B$ $\therefore P = HA$ and $Q = HB$
>
> $\therefore$ $P \pm Q = H(A \pm B)$; and $MP \pm NQ = H(MA \pm NB)$
>
> $\therefore$ The H.C.F. of P and Q is also the H.C.F. of $P \pm Q$, $2P \pm Q$, $P \pm 2Q$ and $MP \pm NQ$

Taking P and Q as positive integers with $P > Q$ we can therefore **proceed by subtracting any multiples of *Q* we like from *P***, and the result will still contain H as a factor. Usually, we find it best tosubtract as many as possible.

Example 1 Find the HCF of 483 and 357.

We subtract one 357 from 483 and note the remainder: 483, 357, 126.

Next, we take 2 126s from 357 and note the remainder: 483, 357, 126, 105.

Then, take 105 from 126: 483, 357, 126, 105, 21.

Here we can stop and declare that 21 is the HCF, because the next step would leave a remainder of zero (105 – 5×21 = 0).

This process, in which the numbers and remainders sequentially create further remainders, comes

ISSN (Online): 2347-601X and Website: www.ijemhs.com
under the Sutra *The Remainders By the Last Digit*

We can make use of any of the remainders that appear, so we may not need to continue right to the end.

Example 2 Find the HCF of 469 and 413.

The sequence begins with 469, 413, 56.

And noting that 56 = 8 × 7, and that 8 contributes nothing to the HCF (since it cannot be a factor of either number), we may test to see if 7 divides into both the given numbers. And since it does, we have HCF = 7.

Or, of course, we can carry on with the repeated subtractions instead.

Example 3 Find the HCF of 42 and 24.

We can only take one 24 from 42, so we get 42, 18 and then 42,18,6. So, 6 is the HCF.

However, taking two 24s from 42 instead of just one leaves a remainder of -6, and though this is negative that result can be used instead.

That way we get the result immediately: 42, 24, -6. HCF = 6

This is easier and can always be applied. That is, we can always subtract the nearest multiple of a number: the remainder does not have to be positive.

HCF of Three or More Numbers

To find the HCF of three numbers, we find the HCF of two of them, as shown above, and then find the HCF of the HCF just found together with the third number. Similarly, for more than three numbers.

Example 4 Find the HCF of 483, 357 and 392.

From Example 1 we know the HCF of 483 and 357 is 21.
We therefore subtract as many 21s from 392 as possible and get a remainder of 14: 392, 21, 14.

Then continuing, 392, 21, 14, 7, and since the next remainder would be zero we can say that the HCF of 483, 357 and 392 is 7.

2.3. HCF of Polynomial Expressions

The same method can be applied here.

Example 5 Find the HCF of $2x^3 + 5x^2 - 11x + 4$ and $2x^3 - 9x^2 + 10x - 3$.

For convenience we may put only the coefficients of the expressions:

2, 5, -11, 4	(1)	this is the 1st expression
2, -9, 10, -3	(2)	this is the 2nd expression
0, 14, -21, 7	(3)	this is (1) minus (2)
0, 2, -3, 1	(4)	we can divide out the factor of 7*
2, -3, 1	(5)	move (4) to the left (this is equivalent to multiplying (4) by *x*)
0, 8, -12, 4	(6)	this is (1) minus (5)
0, 2, -3, 1	(7)	divide by 4

ISSN (Online): 2347-601X and Website: www.ijemhs.com

Since line (7) is a repeat of line (4) we can say the HCF = $2x^2 - 3x + 1$.

* since 7 is not a factor of the given expressions.

4. LOWEST COMMON MULTIPLE, LCM

There are various methods for getting the LCM, some of which involve finding the HCF first.

LCM of Two Numbers

This is easily found from the well-known result that the
(LCM of *P* and *Q*) × (HCF of *P* and *Q*) = *PQ*. (1)

Example 6 Find the LCM of 483 and 357.

Using the above result, (1), and noting that the HCF of 483 and 357 is 21 (see Example 1):LCM of 483 and 357 = (483 × 357) / 21
= 23 × 357
= 8211.

LCM of Three or More Numbers

The above formula, (1), can be extended. For three numbers *P*, *Q*, *R* it would be:
(LCM of *P*, *Q* and *R*) × (product of HCFs of *P*, *Q* and *Q*, *R* and *P*, *R*) = *PQR* × (HCF of *P*, *Q* and *R*). (2

)Example 7 Find the LCM of 60, 70 and 84.

We find the HCF of 60, 70, 84 to be 2 (see section 2.2).

We find the HCFs of 60, 70 and 70, 84 and 60 and 84 to be 10, 14 and 12 respectively.

Therefore using (2) above, the LCM of 60, 70 and 84 = (60×70×84 × 2) / (10×14×12).

Cancelling we find LCM of 60, 70 and 84 = 420.

However, this method becomes a bit unwieldy for more than three numbers, and we may prefer an alternative method.

Alternative Method for LCM of Three or More Numbers

In this method we take any one of the numbers (usually it is best to choose the number with the most factors) and multiply it by those factors in the other numbers that are not already in the chosen number.

Example 8 Find the LCM of 60, 84, 70 and 54.

We can start with 60.

We note that 84 has the factor, 7, that is not contained in 60. So we have 60×7.
Now look at the next number, 70. It contains no new factors.
Then looking at the last number, 54, this contains two more factors of 3. That is, 60 has only one

factor of 3 and 54 has three factors of 3. So, we need to have a further factor of 3×3 = 9. This brings us to 60×7×9 = 3780.

So, the LCM of 60, 84, 70 and 54 is 3780.

This method is good if the numbers involved are quite small.

General Method for LCM

Perhaps the best general method is to use the first method repeatedly.

Example 9 Find the LCM of 60, 84, 70 and 54.

We begin by finding the LCM of 60 and 84.
We find the HCF of 60, 84 is 12. So, the LCM of 60, 84 is (60×84)/12 = 420.

Next, we find the LCM of 420 and 70, and since 70 clearly divides exactly into 420 we can take 420forward as the LCM of 60, 84, 70.

Next, we find the LCM of 420 and 54.
We find the HCF of 420, 54 is 6. So, the LCM of 420, 54 is (420×54)/6 = 3780.

3.5. LCM of Polynomial Expressions

As with the HCF above, we can use the same methods as for numbers.

Example 10 Find the LCM of $2x^3 + 5x^2 - 11x + 4$ and $2x^3 - 9x^2 + 10x -$

3.

This is example 5 except we are now finding the LCM.

The LCM will be the product of the given polynomials divided by the HCF which was found to be $2x^2 - 3x + 1$ in Example 5.

As with the numerical examples it is best to divide by the HCF before multiplying:

LCM of $2x^3 + 5x^2 - 11x + 4$ and $2x^3 - 9x^2 + 10x - 3$ is

$$(2x^3 + 5x^2 - 11x + 4)(2x^3 - 9x^2 + 10x - 3)/(2x^2 - 3x + 1) = (x+4)(2x^3 - 9x^2 + 10x - 3)$$
$$= 2x^4 - x^3 - 26x^2 + 37x - 12.$$

Since we know that the HCF divides into both the given expressions the division of $(2x^3 + 5x^2 - 11x + 4)$ by $(2x^2 - 3x + 1)$ here is easily carried out using *The First By the First and the Last By the Last*: $2x^3 \div 2x^2 = x$, and $4 \div 1 = 4$. Hence $x + 4$.

5. CONCLUDING REMARKS

The same process of repeated division by remainders shown here for the HCF can also be used to convert fractions to continued fractions and conversely[2].

References

[1] Bharati Krishna Tirthaji Maharaja, (1965). Vedic Mathematics. Delhi: Motilal Banarasidas.

[2] Williams. K. R. (2021). Continued Fractions using the *Śeṣa* Sūtra. Online International Journal ofVedic Mathematics