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The Ethics of Mathematics: Does It Cause Harm to Society?

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ABSTRACT

In this paper I challenge the idea that mathematics is an unqualified force for good. Instead I show the harm that learning mathematics can inadvertently cause unless it is taught and applied carefully. I acknowledge that mathematics is a widespread force for good but make the novel case that there is significant collateral damage caused by learning mathematics. I describe three ways in which mathematics causes collateral damage. First, the nature of pure of mathematics itself leads to styles of thinking that can be damaging when applied beyond mathematics to social and human issues. Second the applications of mathematics in society can be deleterious to our humanity unless very carefully monitored and checked. Third, the personal impact of learning mathematics on learners' thinking and life chances can be negative for a minority of less successful students, as well as potentially harmful for successful students. I end with a recommendation for the inclusion of the philosophy and ethics of mathematics alongside its teaching all stages from school to university, to attempt to reduce or obviate the harm caused; the collateral damage of learning mathematics.

Keywords: critical mathematics education, ethics, collateral damage, harm, instrumentalism, philosophy of mathematics

Introduction

Mathematics is a rich and powerful subject, with broad and varied footprints across education, science, culture and indeed throughout all of human history. Both the academic world and society in the large accord mathematics a high status both as an art and as the queen of the sciences (Bell 1952). Mathematics has a uniquely privileged status in education as the only subject that is taught universally and to all ages in schools. Hidden behind this elevated status is the assumption that mathematics is an unqualified force for good. Is this ethical assumption correct? Does nothing but good flow from mathematics? In this chapter I argue that mathematics does harm as well as good. My claim is that mathematics in school has unintended outcomes in leaving some students feeling inhibited, belittled or rejected by mathematics. In sorting and labelling learners and citizens in modern society, mathematics reduces the life chances of those labelled as failures or rejects. In addition, even for those successful in mathematics, the discipline serves as a training that shapes thinking in an ethics-free and amoral way. Thus mathematics supports instrumentalism and ethics-free governance. This is exploited in warfare, heartless corporate activity, the misuse of humans and the environment, and in many acts that treat persons as objects rather than moral beings, entitled to respect and dignity. I conclude by suggesting solutions. To avoid or remedy the negative effects in schooling we need to attend more closely to the causes of success and failure, and become fully aware of how these have far-reaching impacts on learners. Further, in order to forestall the harmful effects of mathematics in society we need to teach the social responsibility of mathematics, through including philosophy and especially the ethics of mathematics alongside mathematics itself. All students of mathematics and fully fledged mathematicians should be able to view the uses and applications of mathematics critically, seeing the mathematics in play and understanding the ethical implications of the issues involved.

As a mathematician myself, someone who has devoted his professional life to furthering the teaching of mathematics in school and university, I might be expected to be among the last to question the value of mathematics. However, I believe that both the place of mathematics in the world and the benefits it brings are strengthened through looking at mathematics as a critical friend. This entails not only lauding and feeling pride at the benefits mathematics brings but also recognising the harm it can do, and not shying away from it. Acknowledging that there are negative outcomes opens the doors to solutions, to possible means of ameliorating and rectifying the possible damage brought about through mathematics.

Is Mathematics an Untramelled Good?

In this chapter I wish to challenge the myth that mathematics is an untramelled good, and that promoting and learning mathematics leads solely to beneficial outcomes and never causes harm. The received wisdom dominating the institutions of mathematics, mathematics education and society in general, is that mathematics of itself is a wonderful boon for all of humankind, and in areas where its positive benefits are not remarked it is simply neutral (Gowers n. d.). Even stronger, Burnyeat (2000) argues that studying mathematics is good for the soul, basing his claims on the arguments of Plato. By contrast, a web searches linking mathematics to harm or damage reveals nothing that challenges the claim mathematics is an untramelled good.[1]

In place of the generally uncritical plaudits that mathematics receives I wish to ask what are or might be the actual outcomes and potential costs of elevating and privileging mathematics in education and society, including any unintended outcomes? Looking at such outcomes, does mathematics cause any harm or evil? To mathematicians and many others even asking this question, let alone answering it in the affirmative, might seem unthinkable, a ridiculous questioning of what has hitherto been unquestionable. To educationists it is not so difficult to ask this question, or even to answer it in the affirmative, when the impact on disadvantaged students and society is considered (Stanic 1989).

Before I address the potential harm that mathematics may do, let me begin by affirming that mathematics has great value. The overall value of mathematics comprises the benefits and goods it offers to humanity as a whole. There are two types of value that mathematics possesses. First, there is the intrinsic value that mathematics has as a discipline or area of knowledge, the value of mathematics purely for its own sake. My claim is that mathematics is one of the great intellectual products of human culture. Thus teaching mathematics is enabling learners to encounter and engage with this great cultural product. Second, there is extrinsic value, the general social value of mathematics on the basis of its applications and uses in society. It is the language of all scientific and technological achievements. It is a

[1] The one exception that I have found lies in feminist critiques of mathematics as oppressive and patriarchal, see, e.g., Burton (1995) and Shelley (1995).

universal language that allows us to understand and share our understanding of all physical forces from the sub atomic to the cosmic. Teaching about this aspect of mathematics opens up the world of mathematical applications to learners allowing them to appreciate its immense practical power as well as to participate in using such applications themselves. In addition to the social benefits of its applications mathematics also has personal value. This is the value of mathematics for learners and for other persons more widely as it plays out in terms of individual benefit. Such benefits will vary across individuals according to personal circumstances, experiences, social contexts and so on. For many students the learning of mathematics results in great personal power, manifested in increased social, professional and study opportunities, as well as enhanced feelings of mathematical self-efficacy and overall self-worth.

The intrinsic value of mathematics

Mathematics has intrinsic value, and as I argue elsewhere the furthering of mathematics for its own sake is an ethical good for humankind (Ernest 2016b). Mathematics expands the human intellect, broadening our conceptual horizons and opening up vast areas of pure thought. Mathematics is a powerful exploration of pure thought, truth and ideas for their intrinsic beauty, intellectual power and interest. In its development mathematics creates and describes wondrous worlds of beauty, populated by linked crystalline forms that stretch off to infinity in richly etched exquisiteness, like the vision of the net of Indra. In addition to their intrinsic value, these forms make up the language of structure that frames virtually all possible abstract conceptual relationships, including those of the sciences and computing. Part of the intrinsic value of pure mathematics is its widely appreciated beauty (Ernest 2016a). "Like painting and poetry mathematics has permanent aesthetic value" (Hardy 1941: 14). "Mathematics possesses not only truth, but supreme beauty – a beauty cold and austere, like that of sculpture" (Russell 1919: 60).

These virtues and values are not only appreciated by those initiated into the most exclusive inner sanctum of mathematics, the area occupied by the ground-breaking creative mathematicians. They also elicit wonder from the public. We are often confronted with complex and fascinating mathematics-based images in the media, for example multi-coloured pictures of fractals, complex tessellations and other beautiful representations. These contribute to the public perception that mathematics can be both beautiful and intriguing, and has an intrinsic value.

The Extrinsic and Social Value of Mathematics

First, with regards to science, mathematics is known as both the queen and servant of science (Bell 1952). As its servant mathematics provides the language through which modern science is formulated. Models, laws, theories and predictions, going back 2000 years ago to the Ptolemaic model of the universe, could not be expressed without mathematics. Since the industrial revolution, scientific applications based in mathematics have underpinned engineering, technology and the whole material basis of modern life.

Second, computing and the information and communication technologies that form the language and basis for all our modern media, knowledge systems and control mechanisms, rest solely on mathematics and logic. Both the knowledge representations and the programmed instructions upon which information and communication technology depends can only be expressed by means of the coding and logic supplied by mathematics.

Third, and far from least, finance, economics, trade, business, and through them, social organisation, rest on a mathematical foundation. Money, the intangible embodiment of economics, is the lifeblood that circulates throughout these bodies and activities. The commercial basis of modern society simply would not be possible without money, and thus, without arithmetic. Money is number that utilises one possible type of unitisation, a quantification of exchange value. This is not surprising given the evidence that tax, tribute and trade and the associated needs for systematic recording is what gave birth to written mathematics five thousand years ago (Høyrup 1994).

Each of these domains of application has undoubtedly many great benefits in terms of human flourishing, including improvements in health, nutrition, housing, transport, agriculture, manufacturing, education, leisure, communications and wealth. More human beings than ever live longer, healthier, better educated, more comfortably and wealthier as a consequence of the mathematics-led developments in the sciences, technology and engineering, especially over the past two centuries.

In addition to these social benefits shared by so many, mathematics has great personal value. Learners and persons in general benefit from mathematics as:

1. an enlarging element of human culture,
2. a means of personal development and growth,
3. a valuable tool for use socially, both as workers, and citizens in society
4. a means of gaining certification for entry to employment or further education.

We live in a mathematized social world. The immense utility of mathematics must be acknowledged as a great strength and virtue. Without it not only would we have to forego many of the tools we as individuals and society rely on, but many of the necessities of life we enjoy and much of our prosperity would disappear. Mathematics is arguably the most generally applicable of all human knowledge fields and many if not most of the good qualities of modern living depend on it.

Features and characteristics of mathematics

An immediate question is what are the components of mathematics that contribute to its great intrinsic and extrinsic value? The most obvious dimensions are that of number and calculation. Calculation is central to mathematics, and it dominates both history and schooling. Mathematics as a scientific discipline is claimed to originate around 3000 years BCE (Høyrup 1980). Thus it was already halfway through its history, around 500 years BCE, before proof entered into mathematics. Prior to that number recording and calculation, plus some geometric measurement, constituted pretty much the totality of mathematics. Even since then, numbers and calculation have dominated both the practical uses of mathematics and its educational content, with Euclidean geometry playing a minor role, and that just in elite education.

At the heart of calculation are rule-based general procedures. In these, the overall meaning of numerals, especially the place-value meaning signified through the relative positioning of the constituent digits, is largely ignored during most of the algorithmic processes. Further, largely as a result of Islamic contributions, algebra emerged in the Middle Ages. This provides the abstract language of mathematics upon which all modern developments depend. Algebra is generalized arithmetic in origin and as such is subject to generalized arithmetical procedures and rules, and its strength is that specific numerical meanings are detached. This was explicitly noted over 300 years ago by Bishop Berkeley.

> … in Algebra, in which, though a particular quantity be marked by each letter, yet to proceed right it is not requisite that in every step each letter suggest to your thoughts that particular quantity it was appointed to stand for. (Berkeley 1710: 59).

At its heart, algebra is variable based, thus forcing the linguistic move in the language away from specific values and meanings to general rules and procedures concerning variables. This move has some great benefits. It enables the miracle of electronic computing in which mathematical rules and procedures are wholly automated and no reference to or comprehension of the meaning of mathematical expressions is required.

Overall, during the application of algorithms and other permitted procedures in arithmetic and algebra the meaning of expressions can largely be neglected with no detriment to the efficacy of the procedures. Meaning is dispensable.

A further characteristic of school, university and research mathematics is that they are represented in the symbolism and language of mathematics, and this is fundamentally in sentences. Mathematical sentences, although often containing symbols, conform to the usual subject-verb form, or more generally, to the terms-relation form, where a relation is equivalent to a generalised verb. In a detailed analysis Rotman (1993) found that although there is some limited use of the indicative mood, the predominant verb form in mathematical language is the imperative mood. Imperatives are orders that instruct or direct actions either inclusively, such as: let us …, consider …, or exclusively, such as: add, count, solve, prove, etc. Imperatives occur more frequently in mathematics than in any other academic school subject (Rotman 1993; Ernest 1998). In addition, mathematical operations require rigid rule following. At its most creative mathematics allows choices among multiple strategies and representations, but each of the lines of choice pursued involves strict rule following. Consequently mathematics is very unforgiving. There is no redundancy in its language and any error in rule following derails the procedures and processes. Thus students of mathematics must learn to use its language and follow its rules with great precision. The net result of extended exposure to and practice in mathematics is a social training in obedience, an apprenticeship in strict subservience to the text, be it printed or spoken. Mathematics is not the only subject that plays this role but it is by far the most important in view of its imperative rich and rule-governed character. Furthermore, the rule following is done without any need for attention to the meaning of the signs being worked on and transformed.

One of the most important ways that a social training in obedience is achieved is through the universal teaching and learning of mathematics from a very early age and throughout the school years. The central and universal role of arithmetic in schooling provides the symbolic tools for quantified thought, including not only the ability to conceptualize situations quantitatively, but a compulsion to do so. This compulsion first comes from without, but is appropriated, internalized and elaborated as part of the postmodern citizen’s identity. We cannot stop calculating and assigning quantified values to everything, in a society in which what matters is what *counts* or is *counted*.

The teaching and learning of mathematics in schools, and thus the development of mathematical identity requires that, from the age of five or soon after, depending on the country, children will (Ernest 2015):

1. Acquire an object-oriented language of objects and processes,

2. Learn to conduct operations on and with them without any intrinsic reasons or sense of value, thus operating with deferred meaning,
3. Decontextualise their world of experience and replace it by a deliberately unrealistic and very stylized model composed of simplified static objects and reversible processes,
4. Suppress subjectivity, experiential being and feelings in their mathematical operations on objects, processes and models,
5. Learn to prioritize and value the outcomes of such modelling above any personal or connected values and feelings, and apply these outcomes irrespective of such subjective dimensions to domains including the human "for [your] their own good" (Miller, 1983).

King (1982) researched the mathematics taught and learnt in 5-6 year old infant classrooms. He found that mathematics involves and legitimates the suspension of conventional reality more than any other school subject. People are coloured in with red and blue faces. "A class exercise on measuring height became a histogram. Marbles, acorns, shells, fingers and other counters become figures on a page, objects become numbers" (King, 1982, p. 244). Further, in the world of school mathematics even the meanings of the simplified representations of reality that emerge are dispensable.

> Most teachers were aware that some children could not read the instructions properly, but suggested they "know how to do it (the mathematics) without it." … Only in mathematics could words be left meaningless (King, 1982, p. 244).

In the psychology of mathematics education instrumental understanding, defined as knowing how to carry out procedures without understanding, versus relational understanding, which includes in addition knowing how and why such procedures work, is much discussed as a problem issue (Skemp 1976, Mellin-Olsen 1987). It is no coincidence that what is termed instrumental understanding is also a form of the instrumental reasoning critiqued by the Frankfurt School, and which is discussed in the sequel.

In summary, many procedures on signs are carried out with abstracted or deferred meanings, and many mathematical texts, be they calculations, derivations or proofs, involve the reader following rule-governed sequences or orders. In education. mathematics is the subject most divorced from everyday or experienced meaning, and the objectification and dehumanisation of the subject are a necessary part of its acquisition.

However, I need to qualify these claims. Although mathematical signs and procedures are detached from meaningful referents in the world, engagement with mathematics can create an inner world of meanings. Successful mathematicians work within richly populated conceptual universes that are very meaningful to themselves. Success at mathematics at most levels is often associated with having a meaningful domain of interpretation of mathematical signs and symbols, often within the closed world of mathematics. In addition, applied mathematicians interpret mathematical models in the world around us so in applications meanings are reattached. Likewise, although mathematical language is very rich in imperatives, successful users of mathematics at all levels have certain degrees of freedom available to them, such as which methods and procedures to apply in solving problems, as is acknowledged above.

These qualifications notwithstanding, the study of mathematics instils both the capacity to, and the expectation of, meaning detachment during reasoning and calculative procedures.

Likewise, it prepares its readers to follow the imperatives in the text during the technical and instrumental reasoning involved in mathematics.[2]

Mathematical thinking as detached instrumental and calculative reasoning

My claim is that the linguistic characteristics and moves indicated above have costs, including unanticipated negative outcomes when extended and applied beyond mathematics. For as I have argued, the mathematical way of thinking promotes a mode of reasoning in which there is a detachment of meaning. Reasoning without meanings provides a training in ethics-free thought. Values neutrality and ethical irrelevance is presupposed because meanings, contexts and their associated purposes and values are stripped away and discounted as irrelevant to the task in hand. Furthermore, as I have argued elsewhere, there is a widespread perception of mathematics as timeless, universal and imbued with absolute certainty, and hence it is viewed as an objective, value-neutral and ethics-free domain of thought (Ernest 1998, 2016a, 2016b). Such reasoning and perspectives contribute to a dehumanized outlook. For without meanings, values or ethical considerations reasoning can become mechanical and technical and 'thing' or object-orientated. These modes of thinking foster what have been termed separated values.

Gilligan (1982) proposes a theory of separated and connected values that can usefully be applied to mathematical and other types of reasoning. Her theory distinguishes separated from connected values positions and places them in opposition. The separated position valorises rules, abstraction, objectification, impersonality, unfeelingness, dispassionate reason and analysis, and tends to be atomistic and thing-centred in focus. The connected position is based on and valorises relationships, connections, empathy, caring, feelings and intuition, and tends to be holistic and human-centred in its concerns. These two value positions can be seen as oppositions, with separated values (first) contrasted with connected values (second, respectively), providing the following oppositional pairs: rules vs. relationships, abstraction vs. personal connections, objectification vs. empathy, impersonal vs. human, unfeeling vs. caring, atomistic vs. holistic, dispassionate reason vs. feelings, analysis vs. intuition.

The separated values position applies well to mathematics. Mathematical objects are entities resulting from objectification and abstraction and are naturally impersonal and unfeeling. Mathematical structures are constituted by abstract and rule-based sets of objects and their structural relationships. The processes of mathematics are atomistic and object-centred, based on dispassionate analysis and reason in which personal feelings play no direct contributing part. Thus separated values fit mathematics very well and indeed can be said to be an essential part of mathematics. Mathematics both embodies and transmits these values.

Separated values and the associated outlooks are necessary, indeed essential, by the very nature of mathematics, and their acquisition constitute assets and are undoubtedly beneficial for thinking in mathematics. A separated scientific outlook is also useful in reasoning in other inanimate domains, such as in physics and chemistry, where atomistic analysis, strictly causal relationships and structural regularities yield high levels of knowledge. However, thinking exclusively in the separated mode can lead to problems and abuses when applied outside mathematics and the physical sciences to society. In the human sphere exclusively separated values are unnecessary and potentially harmful, since they factor out the human and ethical dimensions. In seeing the world mathematically, the richness of nature and human worlds,

[2] In addition, in more advanced study of mathematics in high school or university, students learn to reason and draw inferences from assumptions and postulates that are not necessarily true. Such hypothetical reasoning adds yet another level of detachment from the world we live in, weakening the bonds to reality, values and ethics.

with all their beauty, contextual complexity and linkages, and ethical responsibilities, are replaced by simplified, abstracted and objectified structural models. The outcome parallels Wilde's (1907: p. 116) dictum about the outlook "that knows the price of everything and the value of nothing". Although mathematical perspectives and models are powerful and useful tools for actions in the world, including the improvement of human life conditions, when overextended they risk becoming a threat to our very humanity. Inculcating these values can lead to a dehumanized outlook if applied to social and human worlds. Furthermore, separated values extended too far beyond mathematics can also lead to the view that mathematics and its applications have no ethical or social responsibility. While there are legitimate philosophical arguments that pure mathematics is ethically neutral, although I argue the opposite (Ernest 2016b), it is near universally agreed that mathematical applications bear full social responsibility for their impacts on the world, just as do the applications of science and technology.

My claim is that subjection to mathematics in schooling from halfway through one's first decade, to near the end of one's second decade, and beyond if one so chooses, structures and transforms our modes of thought in ways that may not be wholly beneficial. I do not claim that mathematics itself is harmful. But the manner in which the mathematical way of seeing is integrated into schooling, society and above all into the interpersonal and power relations in society results in the transformation of the human outlook. This is a contingency, an historical construction. It results from the way that mathematics has been recruited into systems thinking instead of empathising (Baron-Cohen 2003) and separated values instead of connected values (Gilligan 1982), that dominate western bureaucratic thinking. It also results from the way mathematics serves a culture of objectification, termed a culture of *having* rather than *being* by the critical theorist Fromm (1978).

One framework that acknowledges these aspects of the application of mathematics is the critique of instrumental reason and rationality provided by the critical theory of the Frankfurt School. Instrumental reason is the objective form of action or thought which treats its objects simply as a means and not as an end in itself. It focuses on the most efficient or most cost-effective means to achieve a specific end, without reflecting on the value of that end (Blunden n. d.). Instrumental reason has been subjected to critique by a range of philosophers from Weber to Habermas (Schecter 2010). This includes Heidegger, who argues that instrumental reason and what he terms *calculative thinking* lead us into enclosed systems of thought with no room for considering the ends, values and indeed ethical dimensions of our actions (Haynes 2008). As Heidegger puts it, even "the world now appears as an object open to the attacks of calculative thought" (Dreyfus 2004: 54). The central argument that means must never trump or eclipse ends, when human beings are the ends, can be found in Kant's *Grounding for the Metaphysics of Morals* of 1785. There he derives his Categorical Imperative from first principles, with the following as one of his conclusions. "Act in such a way that you treat humanity, whether in your own person or in the person of any other, never merely as a means to an end, but always at the same time as an end." (Kant 1993: 36)[3]

[3] This chapter is intended as a contribution to the philosophy of mathematics education and does not summarise the range of ethical theories positions available beyond those of Kant (and the Critical Theorists). These philosopher provide a powerful basis for my argument, but others can also be cited. For example, Emmanuel Levinas is another philosopher who argues that we must treat other humans with infinite respect. According to Levinas another person, the 'Other', is infinitely complex and not fully knowable or reducible to an object that can be known (Levinas 1978).

A broader-based critique comes from the Critical Theorists of the Frankfurt School (including Adorno, Fromm, Habermas, Horkheimer and Marcuse) who see instrumental reason as the dominant form of thought within modern society (Bohman 2005, Corradetti n. d.). By focussing on technical means and not on the ends of their actions, persons, governments and corporations risk complicity in the treatment of human beings as objects to be manipulated, in actions that threaten social well-being, the environment and nature. This outlook underpins the behaviours of some governments and multinational corporations in reducing costs and chasing profits without regards for the human costs. Such actions by corporations have been termed psychopathic (Bakan 2004). We are now so used to the economic, instrumental model of life and human governance that most persons see it as an unquestionable practical reality, a necessary evil, and are not shocked or outraged by corporations or governments treating persons as objects with no concern for their well-being.

Much of the Frankfurt School critique was prompted by the rise of Nazism in Germany, with its authoritarian leaders (Adorno *et al*. 1950) and the heartless complicity of ordinary citizens in Germany and occupied territories before and during World War 2. The capture, transportation, enslavement and murder of millions of fellow citizens was not simply undertaken by monsters. These wholesale activities would not have been possible without many ordinary citizens unquestioningly doing their everyday jobs as part of this monstrous programme. Arendt (1963) terms this ordinariness, from the actions of Eichmann downward, the 'banality of evil'. The fact that many ordinary citizens were highly educated did not prevent them from complicity in mass murder. As Dr. Haim Ginott, a school principal who survived a Nazi concentration camp, wrote in his advice to his teachers:

> I am a survivor of a concentration camp. My eyes saw what no man should witness: gas chambers built by learned engineers, children poisoned by educated physicians, infants killed by trained nurses, women and babies shot and burned by high school and college graduates. So I am suspicious of education. My request is: help your students to become human. Your efforts must never produce learned monsters, skilled psychopaths, educated Eichmanns. Reading, writing and arithmetic are important only if they serve to make our children more humane. Ginott (1972: 317)

My argument is that mathematics plays a central role in normalizing instrumental and calculative ways of seeing and thinking. From the very start of their education children are schooled in these ways of seeing and being. As I have argued, the detachment of meaning and the following of imperatives in mathematical texts provides the central platform for instrumental thought.[4]

There is a further factor too. Among philosophers, mathematicians, as well as in school and more generally, in society, mathematics has the image of objectivity, of unquestionable certainty, with claims being settled decisively as either true or false as well as being ethically neutral (Ernest 1998, Hersh 1997). Thus a training in mathematics is also a training in accepting that complex problems can be solved unambiguously with clear-cut right or wrong answers, with solution methods that lead to unique correct solutions. Within the domain of pure mathematical reasoning, problems, methods and solutions may be value-free and ethically neutral. But carrying these beliefs beyond mathematics to the more complex and

[4] Of course the right social circumstances are needed too. A society with values of strong social-conformity and a culture of obedience to authority is needed, as Milgram (1974) showed in his experiments. However, as I have argued, subjection to thousands of hours of school mathematics and schooling in general will contribute to this.

ambiguous problems of the human world leads to a false sense of certainty, and encourages an instrumental and technical approach to daily problems. This is damaging, for when decision making is driven purely by a separated, instrumental rationality, then ethics, caring and human values are neglected, if not left out of the picture altogether. Kelman (1973) observes that ethical considerations are eroded when three conditions are present: namely, standardization, routinization, and dehumanization. Since mathematics is the essence of instrumental reason, with its focus on means to ends and not on underlying values, and its procedures require standardization, routinization, and dehumanization, the concomitant erasure of ethics is no surprise. Thus a training in mathematical thinking, when misapplied beyond its own area of validity to the social domain, is potentially damaging and harmful.

Qualifying the critique of instrumental thinking

However, I need to qualify the above critique of instrumental thinking and the role mathematics plays in it, so as not to be one-sided in my evaluation. It would be naive not to recognise that not only are reason and rationality essential for the fair running of complex modern societies, but also that depersonalized and objectivised thinking is necessary for all modern management. Complex modern societies and institutions cannot be run ethically or fairly, let alone effectively, without abstracted, depersonalized and objectivised thinking. Central to modern governance is the accumulation, allocation and distribution of resources. Whatever the political and ideological orientation of a government it needs to calculate where resources will most benefit society according to their values. It is the essence of democracy that the priorities for the distribution of resources varies with different elected governments. Whatever are the priorities and values of legitimate governments, and the social goals which are aspired to, resources need to be allocated to fulfil these goals effectively. We would not be able to pursue the practical meaning of our ethics and principles without working out their rational implications. In addition, systematic and rational record keeping is another necessary for fairness and equity. Thus calculative reasoning and instrumental thinking in the service of societal values and goals is a modern necessity. However, this is not an alibi for the blind following of orders from ‘above’. In a good society there must always be a place for ethical objections and whistle-blowing, where individual conscience can be exercised, when values or laws are transgressed, or unfairness or injustices are perceived.

Above I emphasized the values of connectedness and caring in contrast with separatedness (Gilligan 1982) and rationality, because of their absence from mathematics. However, when it come to the fair running of society impersonal reason and rationality are essential. Only giving benefits to those one cares about or empathises with leads to inequality, favouritism and nepotism. As Pinker (2012) argues, against the dictum of the 1960s, love is not all you need, if you seek to be fair or just. In a just society you must treat strangers and other citizens as having equal rights and deserving equal treatment irrespective of any personal feelings towards them. This is also the basis of all legal systems. The law is based on a set of principles or precedents from which applications are deduced or reasoned in its practical applications to cases. Fairness and impartiality of reasoning underpin the dispensation of justice in the courts. Thus depersonalized objectivised thinking is necessary for all modern management of resources, human or material, within an overall framework based on ethical principles. It also underpins the law. Thus the rational and impersonal reasoning inculcated through mathematics makes a positive contribution to a just and fair society.[5]

[5] Note that the terms justification and justice have the same roots. From the 14th century CE on justification has meant the action of justifying and the administration of justice, and justice is the quality of being fair and just – the exercise of authority in vindication of what is right (Harper n. d.). Justification draws on rationality and impersonal reasoning, which therefore cannot be decoupled from values and fairness.

The social impacts of mathematics and its application

One of the key areas where instrumental modes of thinking are widespread lies within the applications of mathematics. I have described some of the broad range of applications of mathematics in society and their widespread benefits. Alongside these beneficial outcomes it is also possible to use mathematics in ways that are hurtful or harmful. My argument is that applied mathematicians should endeavour to be aware of the uses to which their applications are put, and if these are potentially hurtful or harmful should at least consider the consequences and their own involvement as facilitators. Applied mathematicians should assume some responsibility for the applications and technological innovations they help to create. It has been suggested that there should be a Hippocratic Oath for mathematicians (Davis 1988). Given the widespread views of the neutrality of mathematics, even of applied mathematics, this would seem to be an unlikely development. Although there is a British Society for the Social Responsibility of Science, and even a group called Radical Statistics concerned with social responsibility in statistics, there is no society for the social responsibility of mathematics (pure or applied). Indeed the very idea of the social responsibility of pure mathematics will seem to many a contradiction in terms.

There is an outstanding use of mathematics that is not often counted among its applications. This is the role of mathematics as basis of money and finance. Money and thus mathematics is the tool for the distribution of wealth. It can therefore be argued that as the key underpinning conceptual tool mathematics is implicated in the global disparities in wealth and life chances manifested in the human world. It is not an exaggeration to claim that many current forms of capitalism distort equality in and across global societies to the detriment of social justice, as well as promoting consumerism. Of course this is a hot political issue. My argument is not that we should oppose the western capitalist system like the Anti-Globalization and Occupy movements (Wikipedia n. d. a, b). In the successful mixed economies of the West well regulated capitalism is the vital source of wealth and meaningful employment, and provides work, goods and the services we rely on for good living. Instead, my proposal is that we should foster an ethical and in particular a critical, social justice oriented attitude towards mathematical applications alongside mathematical skills, so that students and citizens in our democracies can make up their own minds. There is a substantial literature on critical mathematics education that promotes this goal (Ernest 1991, Ernest *et al.* 2016, Skovsmose 1994, Powell and Frankenstein 1997), Furthermore, the idea that our actions should be ethical and, in particular, promote social justice is now mainstream thinking, at least in Europe, for example the European Union Treaty stipulates that it shall promote social justice (European Union, n. d.).

The social impact of the image of mathematics

An indirect way through which mathematics impacts on society and individuals is through its images, which for the purposes of discussion can be divided into social and personal images. Social images of mathematics include public images, including representations in the mass media, such as film, cartoon, pictorial, and computer representations of mathematics and mathematicians. They also include school images which incorporate classroom posters, equipment, textbook, teacher presentations, and school mathematical activities as experienced by the learners. Parent, peer or others' narratives about mathematics also contribute to its social image. Personal images of mathematics include mental pictures, visual, verbal or other mental representations, and can be assumed to originate from past experiences and encounters with mathematics, as well as from social talk and other public representations. Personal images of mathematics comprise both cognitive and affective dimensions and effects. The types of mathematics as portrayed in its images can include research mathematics and mathematicians, school mathematics, and mathematical applications, both everyday and more complex. Social and personal images of mathematics are intimately related, as personal images must be assumed to result from the lived experiences of learning and using mathematics and from exposure to social images of mathematics. Likewise, social images of mathematics are constructed by individuals or groups drawing on their own personal images, which are then represented and made public. Both kinds of image can have implicit elements of which individuals are not explicitly aware. Thus, what is termed the hidden curriculum comprises those accidental or unplanned elements of knowledge representations and learning experiences within the school curriculum, which can include images of mathematics (Ernest 2008).

One widespread public image of mathematics in Western countries, which may extend more widely, is of mathematics characterised as a difficult subject, viewed as cold, abstract, theoretical, ultra-rational, mainly masculine but nevertheless important (Buerk 1982, Buxton 1981, Ernest 1995, Picker and Berry 2000). Mathematics also has the image of being remote and inaccessible to all but a few super-intelligent beings with 'mathematical minds'. For many people the image of mathematics is also associated with anxiety and failure. For example, when Brigid Sewell was gathering data on adult numeracy for the Cockcroft Inquiry (1982) she asked a sample of adults on the street if they would answer some questions on the subject. Half of them refused to answer when they understood the subject was mathematics, suggesting negative attitudes, or even mathephobia (Maxwell 1989). While attitudes to and images of mathematics may have improved in the past few decades, following the increase of student-centred mathematics teaching approaches, a recent review of the literature reports the persistence of negative images and attitudes toward mathematics (Belbase 2010).

Some of the problems associated with widespread social and personal images of mathematics follow from the perceptions that it is a masculine subject, much more accessible to males than females; and that it is a difficult subject only accessible to a small and gifted minority. The effect of these images, coupled with the negative learning experiences reported by some students, is to foster negative personal images of and attitudes to mathematics often incorporating poor confidence, lack of mathematical self-efficacy beliefs, and dislike of and even anxiety with respect to mathematics. One of the contributors to the negative images of mathematics can be the absolutist image of mathematics as objective, superhuman and value-free (Ernest 1995, 1998). For many this contributes to a sense of alienation and exclusion from mathematics (Buerk 1982, Buxton 1981).

However, it needs to be mentioned that in contrast to these problems, for a different and more successful minority this absolutist image is part of the attraction of mathematics. Mathematics can be seen as unchanging, perfect, and a safe haven from the chaos and uncertainties of everyday life, and for this reason and others making it attractive to this successful minority. Thus no simple generalization can express the complex and varied effects of the public images of mathematics. The same dimensions or perceptions of mathematics may simultaneously attract and repel different groups of students.

One of the persistent myths of the twentieth century has been that females are 'naturally' less well equipped mathematically than males (Burton 1990, Rogers and Kaiser 1995). So two of the detrimental effects of images of mathematics that I shall foreground here are first the negative impact on female students following from the masculine image of mathematics. Second, the negative impact of mathematics related experiences and images on the attitudes and self-esteem of a minority, including many girls and women. The problem with these negative impacts is that mathematics is a highly esteemed and valued subject in schools and universities, perhaps even overvalued. Mathematics examinations are used as a sifting and filtration device in society. Life chances and social rewards are disproportionately correlated with success at mathematics, even within many areas of study and work in which mathematics plays little part. Sells (1973, 1978) has termed mathematics the 'critical filter' in determining life-chances.

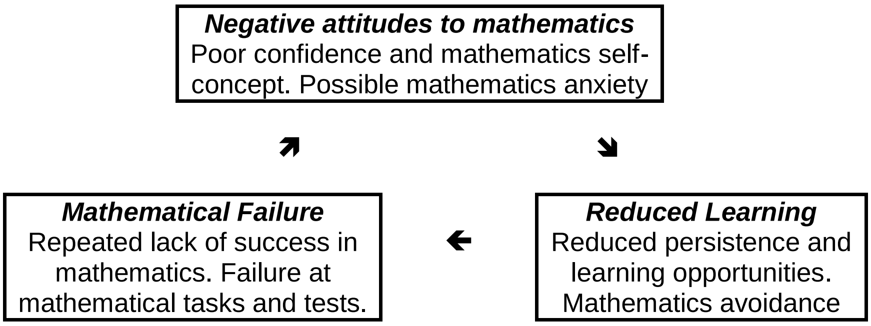
In addition, success in school mathematics is strongly correlated with the socio-economic status or social class background of students. Although this is true with virtually all academic school subjects, mathematics has a privileged status. It is the examinations in mathematics in particular that serve as a fractional distillation device that, to a significant extent, is class reproductive. Talented mathematicians from any background may be successful in life, but the net effect of mathematical examinations is remains the grading of students into a hierarchy with respect to life chances. This hierarchy doubly correlates with socio-economic status and social class, understood in terms of both the social origins and the social destinations of students. So it is not merely raw mathematical talent that is reflected in mathematical achievement. It is also partially mediated by cultural capital (Bourdieu 1986, Zevenbergen 1998).

While mathematical knowledge has important uses and applications in modern societies, the status and value of mathematical achievement is elevated beyond its actual utility. Mathematics is increasingly hidden from citizens in modern society behind complex systems including information and communication technology applications, and the vast computerised control and surveillance systems. These regulate and monitor modern societies for the purposes governance, security and commerce. Advanced mathematical skills are not needed by the many that operate these systems, for such persons can do so successfully without awareness of their mathematical foundations (Niss 1994, Skovsmose 1988). It is the much smaller number of mathematicians, programmers and information technologists that design, implement and test the systems who need advanced specialist mathematical skills.

My claim is that the social image of mathematics as experienced by learners contributes to their personal image of mathematics and that this is an important factor in their success in mathematics. Personal images of mathematics include attitudes to mathematics and these play a key role in success at mathematics via multiplying mechanisms which I call the success and failure cycles (Ernest 2013).

The mechanisms are as follows. Some students suffer from negative attitudes to mathematics, including poor confidence and poor mathematical self-concept, and in a minority possible mathematics anxiety (Buxton 1981). Based on Maslow's (1954) hierarchy of needs theory, it can be said that persons will do a great deal to avoid risks including the risk of failure in a socially esteemed activity, with its concomitant threat to personal self-esteem. So negative attitudes lead to reduced persistence, and even mathematics avoidance in some cases, resulting in reduced learning opportunities. A consequence of this is lack of success in mathematics, which in the strong case is failure. Students who experience an overall lack of success and repeated failure at mathematical tasks and tests develop or strengthen their negative attitudes to mathematics, completing a self-reinforcing cycle, leading to a downward spiral in all three of its components, illustrated in Fig. 1.

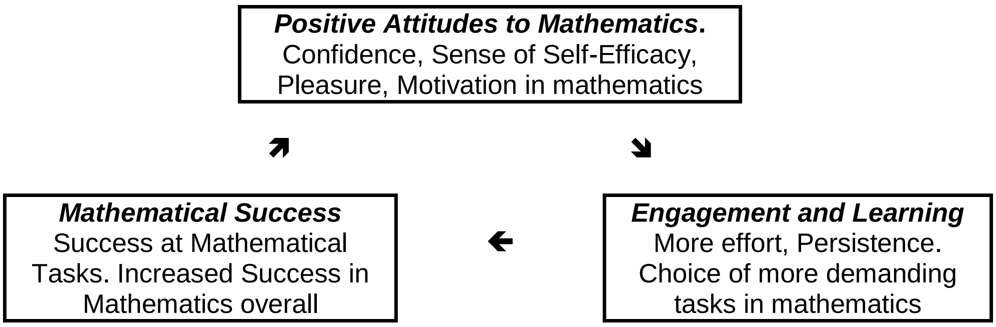
Figure 1: The Failure Cycle (adapted from Ernest 2013).



In this, as in any proper cycle, there is no identifiable beginning point. All three elements develop together, and any one of them could be nominated as a starting point. Thus, outcomes shape attitudes, and in particular failure often leads to poor attitudes. Negative attitudes impact upon behaviours, such as disengagement and low effort. Disengagement in turn reduces the chances of success. So once the cycle is started it becomes self-reinforcing and self-perpetuating, a vicious cycle.

In contrast, positive student attitudes to mathematics, including confidence, a sense of mathematical self-efficacy, pleasure in and motivation towards mathematics lead to increased effort, persistence, and the choice of more demanding tasks. This is because of the intrinsic rewards gained, such as intellectual satisfaction and pleasure gained through success. The increased efforts and engagement in turn lead to students' improved learning, as well as their experience of further success at mathematical tasks and mathematics overall. Consequently, positive student attitudes to mathematics are reinforced, completing a success cycle, in an enhancing upward spiral.

Figure 2: The Success Cycle (adapted from Ernest 2013).



Psychologists including Howe (1990) have shown that a mechanism like that shown in Fig. 2 is an important factor in the development of exceptional abilities among gifted and talented students. Students who demonstrate some giftedness and talent at around the age of 10 are often very significantly further ahead of their peers at the age of 20 precisely because of the factors shown in the figure. Early success and the attitudes it breeds lead to much greater effort, persistence, and choice of more demanding tasks which lead to the flowering of the later manifested exceptional abilities. Howe found that the exceptionally talented invested an extra 5,000 hours in practice of their skills and abilities. This was double the time spent by their capable but less outstanding peers. This finding has been popularized as the '10,000 hour rule' by Gladwell (2008). This rule proposes that 10,000 hours of practice in any activity or skill leads to expertise and mastery.

Figure 2 contains within it practical means of overcoming or ameliorating the failure cycle shown in Figure 1. Students need to be given tasks and support in learning mathematics so that they experience success. This needs to be real success at tasks that fall within their zone of proximal development (ZPD), that is tasks that lie beyond what a learner can do without help, but which are within their grasp with guidance and support from teachers, adults or peers (Vygotsky 1978). Relatively low level tasks that use knowledge and skills that students have already mastered may seem too easy to them, and may not contribute to a sense of success. However, when students experience repeated success at more demanding mathematical tasks, and become used to increased success in mathematics overall, one can expect improved attitudes to mathematics. Over time, students should come to 'own' their success and feel pleasure in it. They will therefore grow in confidence and develop their sense of mathematical self-efficacy.[6] They should gain motivation in engaging in and completing mathematical tasks. Following on from their increased engagement we would expect them to make more effort on mathematical tasks, become more persistent in solving mathematical problems, sometimes even choosing more demanding tasks. Thus they are now following a success cycle (Figure 2). With this in mind the teacher's role is to set tasks that fall within the learner's ZPD, and to offer support and encouragement so they experience success and develop positive attitudes. If a student is firmly in the grip of the failure cycle (Figure 1) it may take some time and significant personal support to initiate the success cycle. But once initiated there should be an upward spiral in terms of gains in attitudes, engagement and success.

Another impact of the social image of mathematics is in sex-differences in mathematical achievement and participation. Traditionally Western females have had lower levels of achievement in school mathematics and lower levels of participation in advanced mathematical study and in mathematical careers than males. Although school level achievements in mathematics have more or less evened out between the sexes in the 21st century, research shows that females continue to have, on average, more negative attitudes to mathematics than males, and this continues to be reflected in continuing lower levels of participation after the age of 16 years (Forgasz *et al.* 2010). It is widely claimed that the social image of mathematics is a significant causal factor in these sex-differences (Mendick 2006). Thus widespread gender stereotyped social images of mathematics include the view that mathematics is a male domain and is incompatible with femininity (Ernest 1995). This contributes to gender stereotyped school images of mathematics which are manifested in a lack of equal opportunities, such as in classroom interactions in learning mathematics (Walkerdine 1988, 1998). Social images, as well as these school factors lead to gender-stereotyping in females' individual images of mathematics and impact negatively on their confidence and perceptions of their own mathematical abilities (Isaacson 1989). The disadvantaging effects of these factors result in underachievement and lower participation rates in mathematics post-school.

[6] Students with positive mathematical self-efficacy attitudes often attribute their success in school mathematics to stable and intrinsic causes such as their own skill and ability, while attributing their failures to extrinsic and unstable causes such bad luck, or a lack of effort (Weiner 1972).

However, in the past two decades, female underachievement has been balanced out by male underachievement due to a separate set of factors, such as many young men's disengagement from school, especially in Anglophone countries such as United Kingdom (Forgasz *et al.* 2010).[7] However, rather than meaning that equality between the sexes has been achieved, it means that there are now two different gender-rated problems related to school mathematics, and that these partially cancel out by negatively impacting differentially on both boys and girls. Furthermore, the lower female participation in higher mathematics post-school remains a significant problem.

Of course I have reported this in a primarily Anglocentric way, and many countries do not follow this pattern. For example in West Indian, Pacific Island states and some Middle Eastern countries girls have been outperforming boys in all subjects, including mathematics. In Latin American countries and Southern European countries the stereotypically male pattern of success in mathematics and science related studies and careers has fallen away. Furthermore, in many Eastern countries mathematical success is seen to be due to student effort and not due to inherited ability, including that associated with sex. However, where such problems persist, as they do in the most populous English speaking countries, images of mathematics are regarded as making a significant contribution.

Summary and provisional solutions

I have critiqued the idea that mathematics is an untrammelled force for good. Instead I offer the metaphor that mathematics has two faces, the good and bad faces. The good face displays the benefits and value of mathematics. I have argued that mathematics is intrinsically a force for good, a creative development of the human spirit and imagination. It is also good in its utility, for it has many benefits in its social applications and personal value that benefit human flourishing. But, more controversially, I also claim that mathematics has a bad face. It does harm through dehumanized thinking which fosters instrumentalism and ethics-free governance. Also, because of its over-valuation in the modern world through education it facilitates social reproduction and the perpetuation of class-based social injustice. Through its social image (coupled with school learning experiences) it aids the development of negative attitudes in some learners, and its gender-biased image maintains social disadvantage for females, especially in the English speaking world.

There are of course, in addition, ethically questionable and harmful applications of mathematics, as there are of any scientific and technological subject. Thus, for example, mathematics, science and technology are used in the manufacture of guns, explosives, nuclear and biological weapons, battlefield computer systems, tobacco products, and other potentially destructive artefacts and tools. But, there is a well known and legitimate argument that it is only in the choice of applications of mathematics in such activities that ethical considerations and violations emerge. My critique is independent of such deliberate applications, and perhaps even precedes them. I question whether mathematics itself, even before its wider applications beyond schooling, is solely a force for good, incapable of detriment and social harm. This view, which I might term a myth, hides the fact that mathematics through its actions on the mind is already implicated in some potentially harmful outcomes even before it is deliberately applied in social, scientific and technological applications.

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[7] This problem is particularly acute for boys from lower socio-economic status groups, who are often less engaged with most academic school subjects including the sciences (Banerjee 2016).

However, some caveats to this argument are required. First of all, from the perspectives that I term absolutist philosophies of mathematics (Ernest 1991), the image of mathematics that I have criticised follows as a necessary feature of mathematics emanating from its very nature. Although I and some others reject the associated absolutist epistemologies and ontologies, these remain legitimate philosophies of mathematics. Secondly, the fact that the mindset fostered by mathematical thinking can lead to harm when it is misapplied to social and other philosophical issues is a defect of human or social thinking, and not an intrinsic weakness of mathematics. Thirdly, such instrumentalist and abstracted modes of reasoning are necessary in modern governance and management, and provided the background values are humane and directed at human flourishing should do no harm. Fourth, the damage done by social images of mathematics is mediated by interpretations of mathematics, that is, socially and personally constructed images of mathematics. These images are not inescapable logical consequences of mathematics itself, for they can, are, and have been different in different societies and at different historical times. Thus the force of my critique is not directed at mathematics itself, but at the social institutions of mathematics, including training in mathematics, and the false social images of mathematics that they can legitimate and project. The harm that I am highlighting comes from what are largely unconscious misapplications of mathematics, including the modes of thought it generates, and from the image of mathematics that many find excluding and off-putting, as well as the current overvaluation of mathematical achievement in school and society.

Thus mathematics is not intrinsically bad or harmful, but as I have argued, its applications, both conscious and unconscious can be detrimental to many. This provokes the question: how can we prevent, ameliorate, or rectify this? In the space here I can only sketch a few possibilities for addressing these problems. I have already sketched how the personal damage done to some via the teaching of mathematics can be ameliorated or rectified. My further proposal is that we should include elements of the philosophy of mathematics and of the ethics of mathematics and its social responsibility in the teaching mathematics at all levels from school to university.

1. Teaching the philosophy of mathematics

My proposal is that we should include selected aspects of the philosophy of mathematics in the school mathematics curriculum and in university mathematics degree courses. Students at all levels should have some idea of proof and how mathematical knowledge is validated. This includes knowing that no finite number of examples can prove a generalisation, whereas a single counterexample can falsify it. Students need to understand the limits of mathematical knowledge, including the following: the certainties of mathematics do not apply to the world, there is always a margin of error in any measurement; no mathematical application or scientific theory can ever be proved true with certainty, and this applies to any mathematical model of the world. Likewise we need to teach the limits of mathematical thinking: the true/false dichotomies we find in mathematics do not apply to the world, where matters are almost never so clear cut. In addition, students need to be aware that there are controversies in the philosophy of mathematics over the nature of mathematics, especially the basis of mathematical knowledge and the status of mathematical objects; that there are controversies over whether mathematical knowledge is absolute, superhuman with an existence that predates humanity, and over whether the objects of mathematics exist in a superhuman Platonic space. A recent issue concerns whether humanly unsurveyable computer proofs, such as that of the 4-colour theorem, are indeed legitimate proofs. Strong disagreements rage over whether mathematics is intrinsically value- and ethics-free or value laden, and over whether it is invented or discovered. I believe that elements of the history of mathematics and mathematics in history can serve to make some of the above recommended points and to humanize mathematics. This can be reinforced by illustrating the ubiquity of mathematics in culture, art and social life. I have just picked out here some philosophical questions and issues that mathematics raises, and many more could be added.

Overall, my proposal is that students should see mathematics as more than just a set of tools, and instead be shown that it is long-standing discipline with its own philosophical issues and controversies, including human and ethical dilemmas. They should learn that mathematics is not an isolated and discrete area of knowledge, which despite having a distinct identity has rich connections with all other dimensions of human activity, practice and knowledge. The importance of grasping aspects of the complex interrelationships between mathematics and the human world is that some of the misunderstandings arising from an isolated and separated view may be obviated. By exploring some of the basic philosophical issues and presuppositions underpinning mathematics, as well the nature, validity and limitations of its knowledge, some of the ills that I have described can be reduced or avoided.

2. Teaching the ethics and social responsibility of mathematics

Although there is a widespread misperception, from my perspective, that mathematics is neutral and bears no social responsibility, clearly its uses and applications are value-laden. We should, in my view, add the ethics of mathematics to all university mathematics degree courses so that mathematicians gain a sense of its social responsibility. We need to teach that mathematics must be applied responsibly and with awareness, and that it is wrong to ignore or label its negative social impacts as ‘incidental’ outcomes or as ‘collateral damage’, thus allowing them to be viewed as outside of the responsibilities of mathematicians. In addition to teaching the ethics of explicit mathematical applications we also need to teach that mathematics has unintended ethical consequences. Thus, we need to teach the limits and dangers of instrumental thinking which mathematics can foster, and how it can lead to dehumanized perspectives in which people are both viewed and treated as objects.

Part of the social responsibility of mathematics is to foster public understanding. Mathematicians, including the wider professional mathematics community, have the responsibility both to promote the understanding of mathematics and to counter misconceptions and misunderstandings about the meanings and significance of its uses and applications in the public domain, especially in the media. Modern citizens need to be critically numerate, able to understand the everyday uses of mathematics in society. As citizens, they need to be able to interpret and critique the uses of mathematics in social, commercial and even political claims in advertisements, newspaper and other media presentations, published reports, and so on. Mathematical knowledge needs to be critical in the sense that citizens can understand the limits of validity of any uses of mathematics, what decisions are conveyed or concealed within mathematical applications, and to question and reject spurious or misleading claims made to look authoritative through the use of mathematics. Citizens need to be able to scrutinize financial sector and government systems and procedures for objectivity, correctness and uncover hidden assumptions. Ideally they should able to identify the ethical implications of applications of mathematics to guard against the instrumentalism and dehumanization that can be hidden behind technical decisions. My claim is that every citizen needs these capabilities to defend democracy and the values of humanistic and civilised societies, and it is part of the social responsibility of mathematics to help provide them. This responsibility begins with school teaching, where all students spend thousands of hours studying mathematics. The critical mathematics education movement has over the past quarter century provided both theoretical analyses and practical examples of what teaching critical mathematics means (D'Ambrosio 1998, Ernest *et al.* 2016, Frankenstein 1990, Skovsmose 1994).

A purist objection to such additional teaching targets is, first of all, that they would steal valuable time and thus detract from the teaching of mathematics, and second that these not the responsibilities of mathematicians. With respect to the first objection it can be said that what I am proposing is not intended to take up even 2% of the time devoted to mathematics teaching in schools and universities. At school, such issues can be brought up within the mathematics curriculum periodically but without taking even a whole lesson. A discussion of examples, models and applications can lead to the issues being raised 'naturally', provided mathematics teachers have been well prepared to do this. Furthermore, using problems concerning the environment, international trade, world development issues, for example, will motivate and engage learners in their mathematical studies as well as highlighting the social responsibility of mathematics. At university a small, time limited course on ethics and social responsibility of mathematics could easily be added as a mandatory course alongside pure, applied or service courses in mathematics. Thus, the costs in time could be very small, meeting this objection, although the positive impacts, in terms of mathematicians' and other mathematics users' awareness of the social responsibility of mathematics, could be significant.

With regard to the second objection on lack of social responsibility of mathematics, it is first interesting to contrast the received views about the responsibilities of mathematics and mathematicians with parallel views about the social responsibilities of science and scientists. Unlike the case in mathematics, there is widespread acknowledgement of the social responsibility of science. Many have argued that what they term the Promethean power of modern science and technology warrants an extended ethic of social responsibility on the part of the scientists and technologists (Bunge 1977, Cournand 1977, Jonas 1985, Lenk 1983, Luppicini 2008, Moor 2005, Sakharov 1981, Weinberg 1978, Ziman, 1998). In particular, The Russell-Einstein Manifesto called for scientists to take responsibility for developing weapons of mass destruction and urged them to "Remember your humanity, and forget the rest" (Russell and Einstein, 1955). This manifesto initiated the Pugwash meetings which emphasised "the moral duty of the scientist to be concerned with the ethical consequences of his (sic) discoveries." (Khan 1988, p. 258). When accepting The Nobel Peace Prize on behalf of himself and the Pugwash conferences Joseph Rotblat stated "The time has come to formulate guidelines for the ethical conduct of scientist, perhaps in the form of a voluntary Hippocratic Oath. This would be particularly valuable for young scientists when they embark on a scientific career." (Rotblat 1995). Thus Rotblat and his colleagues propose that ethics needs to be included in the training of young scientists, a call that is echoed by many others including Bird (2014), Evers (2001) and Frazer and Kornhauser (1986). This call has been taken up authoritatively by UNESCO which emphasizes the theme "Ethics of Science and Technology" (UNESCO n. d.), and according to which "The ethics and responsibility of science should be an integral part of the education and training of all scientists". UNESCO (1999: section 3.2.71). Beyond this, Ziman claims that what is needed is what he calls 'metascience', an educational discipline extending "beyond conventional philosophy and ethics to include the social and humanistic aspects of the scientific enterprise" (Ziman 2001, p. 165). He argues that metascience should become an integral part of scientific training in order to help equip scientists of the future with the skills necessary to tackle ethical dilemmas as they arise (Small 2011).[8]

The situation is rather different in mathematics with the exception of the Radical Statistics group (n. d.), which publishes analyses of social problem topics with the aim of demystifying technical language and promoting the public good. Generally, very few mathematicians acknowledge the ethical and social responsibilities of mathematics, although there is some acknowledgement of the social responsibility of mathematicians, as I recounted above. Hersh (1990, 2007) discusses ethics for mathematicians, Davis (1988) proposes a Hippocratic oath, and the American Mathematical Society (2005) provides Ethical Guidelines for mathematicians. However, the content of these recommendations is primarily about professional conduct in research and teaching for professional mathematicians. Davis (1988) goes beyond this and argues that mathematics should not be put in the service of war or other harmful applications, and mathematicians should exercise their consciences. Ernest (1998, 2007), Davis (2007) and Johnson (2015) argue that mathematics needs to acknowledge its social responsibility, with Davis (2007) arguing for the need for ethical training throughout schooling for mathematicians and non-mathematicians alike. These, however, represent marginal voices in the mathematical and philosophical communities of scholars.

If one looks beyond mathematicians and philosophers to the area of mathematics education, there are many voices asserting the social responsibility of mathematics. Of course it is uncontroversial to claim that education is a value-laden and ethical activity, since it concerns the welfare of students and society, and the objectivity, purity and neutrality of mathematics itself is not at stake. In consequence, there is a very large literature comprising many thousands of publications on social justice and social responsibility in mathematics teaching, the first theme to be mentioned here.[9] Some of the main dimensions in this literature are mathematics and exclusion based on race and ethnic background (Powell and Frankenstein 1997), gender and female disadvantage (Rogers and Kaiser 1995, Walkerdine 1988, 1998), low 'ability' and handicap as obstacles (Ernest 2011, Ruthven 1987), and disadvantages correlated with or caused by social class and its correlated cultural capital or other factors (Cooper and Dunne 2000). A second theme is the role mathematics plays in critical citizenship and the public understanding of mathematics (Frankenstein 1990). A related third theme is the Mathematics Education and Society (Mukhopadhyay and Greer 2015), Critical Mathematics Education (Skovsmose 1994, Ernest *et al.* 2016) and Ethnomathematics (D'Ambrosio 1985, Powell and Frankenstein 1997) movements which consider both the role mathematics plays in society and how it impacts on the first two themes. The Critical Mathematics Education movement also looks critically at mathematical knowledge and the institutions of mathematics and their role in denying the relevance of ethics and values to mathematics, and thus denying its social responsibility (Skovsmose 1994). It shares this concern with the Philosophy of Mathematics Education movement (Ernest 1991, 2016a, 2016b), to which the present chapter and indeed this entire volume represents a contribution. However, within the mathematics education research community, beyond any commitment to the teaching of mathematics in a socially just way, the idea that ethics needs to be taught alongside mathematics remains a minority opinion, except perhaps within research in the third theme distinguished here.

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[8] The inclusion of metascience in science teaching loosely corresponds with my proposal to include the philosophy of mathematics in or alongside the teaching of mathematics.

[9] A very partial bibliography of mathematics education published 20 years ago has over 800 mathematics education entries concerning the issues of society and diversity (Ernest 1996).

Conclusion

In this chapter I question and challenge the idea that mathematics is an unqualified force for good. I acknowledge the traditional argument that like any other instrument, mathematics can be applied in both helpful and harmful ways, and I acknowledge the many benefits it brings. But I nevertheless endorse the minority view that mathematicians and other students of mathematics need to be taught the ethics of mathematical applications to question and limit harmful applications. They also need to be taught to think critically, understanding the uses of mathematics in society and in arguments justifying political claims, social policies and commercial interests. However, my main argument is more radical. I argue that in addition to the explicit and intended applications of mathematics, the nature of mathematical thought and the role mathematics plays in education and society can lead to collateral damage; some unintended but nevertheless harmful consequences. Mathematics has a hidden role in shaping our thought and society that is rarely scrutinised for its social effects and impacts, some of which are negative.

First of all, there is the harm caused by the overvaluation of mathematics in society and education, with its negative impacts on the confidence and self-esteem of groups of student including females and lower attainers in mathematics. These unintended outcomes of mathematics in school in leaves some students feeling inhibited, belittled or rejected by mathematical culture and perhaps even rejected by the educational system and society overall. In sorting and labelling learners and citizens in modern society, mathematics reduces the life chances of those labelled as mathematical failures or rejects (Ruthven 1987). This is a hidden impact of mathematics that is usually brushed over as the fault of the individuals that suffer, rather than as a direct responsibility of the role accorded to mathematics in education and society.

Second, even for those successful in mathematics, in shaping thought in an amoral or ethics-free way, mathematics supports instrumentalism and ethics-free governance. Instrumental thinking leading to the objectification and dehumanisation of persons in business, society and politics, has the potential to cause great hurt and harm. This is manifested in warfare, the actions of psychopathic corporations, the exploitation of humans and the environment, and in all acts that treat persons as objects rather than moral beings deserving respectful and dignified treatment throughout (Marcuse 1964).

I do not claim that mathematics is intrinsically harmful, but that without more careful thought about its role in society and thought it leads to harmful, albeit unintended, outcomes. The way we teach and how we use mathematics and its impact on our thinking is what is harmful. My proposal is that to obviate or prevent the potential harm done by mathematics as well as improving the teaching of mathematics we need to teach the philosophy and especially the ethics of mathematics alongside mathematics itself. Part of this teaching is needed to overcome the idea that mathematics, unlike any other domain of human knowledge bears no social responsibility for its roles in society, science and technology. All human activities should contribute to the enhancement of human life and general well-being and no domain can stand apart from such ethical scrutiny, although this should never be used as a reason for limiting advances within pure mathematics itself. However, the intended and unintended applications of mathematics and their consequences do need to be scrutinised and held accountable within the court of human happiness and human flourishing.

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