International Journal of Engineering, Management, Humanities and Social Sciences Paradigms (Volume 30, Special Issue of Nov 2018)
ISSN (Online): 2347-601X and Website: www.ijemhs.com

An overview of mathematical modelling from a theoretical perspective

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Abstract

Mathematical modelling is one of the cornerstones of mathematics education. A real-world problem is transformed into a mathematical form through the process of mathematical modelling. Modeling entails creating circumstances from real life or transforming difficulties from mathematical explanations into situations that are plausible or realistic. This method asserts that mathematical models play a crucial role in all branches of mathematics, including arithmetic, algebra, geometry, and calculus. As a result, all age groups of higher learning, high school, and primary school may offer mathematical modelling. When new studies on mathematical modelling are studied, modeling's significance becomes clear. The significance, key components, and methodology of mathematical modelling will be discussed in this paper.

Keyword: Mathematics education, mathematical modeling

1. Introduction

Many mathematics education researchers have been recently conducting studies on mathematical modeling in education. Learning and teaching of modeling and its applications involve many aspects of mathematical thinking and mathematics learning (Burkhardt and Pollak, 2006; Niss, 1987, Reported by Mousoulides et al., 2005). The basic reasons that drive mathematics educators to study on mathematical modeling are as follows: the question, "What kind of a mathematics education must be performed in order for students to gain mathematical knowledge and mathematical thinking skills that they can use in real life?" and the concern on the insufficiency of traditional methods and problem solving activities in developing problem-solving skills of the students (Kertil, 2008).
According to Sağırlı (2010); one of the questions, which are frequently asked primarily by the students who are taking mathematics education and the students who are educated in other fields, is "What is the use of this mathematics?" (Özalp, 2006). The fact that the students ask this question is indeed fairly normal. That is because mathematics is generally regarded as an activity that is separate from real life and performed only in schools. However, mathematics is actually a systematic way of thinking that generates solutions to real-life situations and problems via modeling (Niss, 1989), and it is one of the most efficient tools in searching solutions to real-life problems (Özalp, 2006). Mathematical modeling and its applications as well as learning and teaching are followed up by numerous researchers in many parts of the world in order to show the students the role of mathematics in real life (Kaiser, et al., 2006). When the studies on mathematical modeling were examined, it was observed that the students could review and arrange their concept perceptions thanks to the mathematical modeling (Ottesen 2001; Lingefjard 2005). Moreover, the students who took this course in mathematics were more successful in

mathematical modeling skill test (Keskin, 2008). It is observed that mathematical modeling is rather used on higher education levels in teaching a number of courses like analysis and linear algebra. Besides this, it can be stated that mathematical modeling offers examples that belong to real life and increases the attention of the students towards the course. When the other researches were examined, it was observed that they stated mathematical modeling must be included in university education, and suggestions were made on this issue. In some researches, the difficulties experienced by the students during the process of making mathematical modeling work were detected, and it was stated that some students had deeper concept perceptions thanks to mathematical modeling. Researches, which utilized technology in the application of mathematical modeling, were also encountered (Sağırlı, 2010).

Today, the need is increasing towards individuals who are harmonious with technology in engineering, architecture, economy, technology and many other fields and whose skills of problem-solving and making mathematical modeling is developed (Lingefjard, 2006).

There emerges a necessity to develop modeling skills of students by using mathematical modeling approaches in mathematics education. The task of fulfilling this necessity primarily belongs to teachers. If teachers themselves do not have sufficient knowledge on mathematical modeling and have not been able to develop their modeling skills adequately, it can be envisioned that the students will naturally experience significant problems in this course (E. Bukova Güzel, I. Uğurel, 2010).

2. Model and Modeling

Although the terms ‘model’ and ‘modeling’ may seem common terms, they are concepts that incorporate different meanings. Model refers to a product that comes out as a result of modeling whereas modeling refers to a process. Model means to represent the realities belonging to physical life with a number of meaningful symbols and to simplify what is complicated. We may come across models during our daily lives in situations where reflecting the reality is impossible or where access to reality is limited in that particular moment. For instance, an architect can exemplify the features of a building that he/she wants to sell by modeling the building that he/she will construct. By getting models to wear the clothes that they want to exhibit, fashion designers can enable other people to have an opinion on how their clothes look. Children can meet the models of reality (cars, trucks, trains, etc.) in their toys. Many further examples can be given regarding how the models appear in our daily lives. However, there will be two common points in all the examples that will be given. The first common point is that the models are formed to be able to meet the reality or think about the reality. The second common point is that the models are more simplified or more idealized form of some things (Lingefjärd 2007; Reported by Sağırlı, 2010).

One of the most important objectives of mathematics instruction is to earn students the mathematical thinking skill. This skill is significant for students to interpret the situations and generate solutions in a problem situation. Therefore; the models, diagrams and concepts, which exist in their minds or which will be formed by the students to interpret the problem situations that they come across and to generate a solution, are among the subjects that must be emphasized in mathematics education. Mathematical model is a mathematical form like a formula, equation, graph or table that reflects the important features of a given situation whereas mathematical modeling is defined as the process to develop a mathematical model (The Consortium for Foundation Mathematics 2008). Mathematical modeling is the achievement in transforming a situation of the real word into a mathematical problem through the use of a mathematical model. Briefly speaking, mathematical modeling is a simplified representation of the basic characteristics of the real situation through the use of a suitable set of mathematical symbols, relations and functions (Voskoglou 2006).

According toGalbraith and Clatworthy (1990), mathematical modeling is defined as the implementation of mathematics in solving unstructured problems in real-life situations. In these modelings, mathematical approaches are used in finding solutions related to real-life problems. The real-life problem that we come across is transformed into a mathematical problem and solved using mathematical techniques (Cheng, 2001).

According to Gravemeijer, Stephan and Cobb (2002), models emerge as a result of informal activities of the students in the classroom environment. An important development, which must be observed during learning process,

is to reach mathematical models via real-life situations or problem situations. The students will be able to utilize these models in mathematical thinking processes only after these developments. Mathematical model is a concept that also incorporates mental representations and diagrams. Mathematical model is the sum of the structures like equations, functions, graphs and mathematical thinking skills that are existent or formed later in our minds in order to mathematically express a problem situation or a real-life situation. Mathematical models are conceptual tools that are required by individuals to mathematically interpret the problems and situations that they come across (Kertil, 2008).

Berry and Houston (1995) defined mathematical model and mathematical modeling as follows:

- Mathematical modeling provides a method for solving problems mathematically.
- A mathematical model is a mathematical representation of the relationship between two or more variables relevant to a given situation or problem.
- Finding mathematical models is a skill that we hope students will develop in this course.

3. The Importance of Mathematical Modeling

In its broadest sense, mathematical modeling is the process of trying to mathematically define a nonmathematical situation, phenomenon and the relationships between the situations, and finding out mathematical patterns within these situations and phenomena. This definition is the broadest and the most liberal definition of mathematical modeling (Verschaffel, Greer and De Corte, 2002). It incorporates the processes of revealing the relationships, conducting mathematical analyses, obtaining results and reinterpreting the model.

According to Lingefjard (2006), mathematical modeling is a process. In modeling process, there is no necessity to comply with a certain rule during the process of reaching the target using what is given. On the contrary, there are more than one trial-error procedures between what is given and the target in order to reach the solution in modeling process (Lesh ve Doerr, 2003; Blum and Niss, 1991; Crouch and Haines, 2004; Kertil, 2008).

Mathematical modeling is a subject that has been catching the attention of mathematics education researchers in recent years (Mousoulides, et al., 2005). The studies on mathematical modeling and mathematical modeling definitions and approaches that are mentioned in these studies are based on different theoretical foundations (Kaiser et al., 2006). According to Kertil (2008), although the definition and objective of each modeling approach and the way that they are applied in the curriculum differ, some researchers embrace modeling as a paradigm that is beyond constructivism in mathematics instruction, a new approach in mathematics instruction, a number of researchers degrade mathematical modeling to a definition as expressing real-life situations in mathematical language. Consequently, we do not come across with a definition of mathematical modeling that has been put in a certain mold in the literature.

According toKaiser and Sriraman (2006), mathematical modeling is widely used in scientific and technological disciplines. They regard mathematical modeling as applied problem-solving, and oblige real-life criteria for modeling. As a result of their studies on mathematical modeling for many years, Lesh and Doerr (2003) mention model and modeling approach as an alternative to constructivist approach in mathematics instruction. Model and modeling approach emphasizes that not all knowledge can have a structuring process (e.g. mathematical formulae and the rules that will remain in simple knowledge levels), that is to say, some structures do not require structuring, and only one process among many processes that are existent in the mind as a mental activity (e.g. classification, organization) is the structuring process. Greer, Verschaffel and De Corte (2002) took the applications of mathematics as mathematical modeling in order to solve the problem situations that we come across in real life. On the other hand,Lehrer and Schauble differentiated mathematical representation and modeling, and accepted mathematical representation and relationships as a good start in mathematical modeling.

In their study, English and Watters (2004) showed that the modeling activities that they performed with elementary level students developed mathematical thinking skills and problem solving skills of students more than traditional problem-solving activities. Moreover, the results of this study that was performed with elementary school 3rd grade students signified that upper-level mathematical concepts and models can be given to even the students at this level with mathematical modeling activities.

In NTCM (1999), it was stated that mathematical modeling is quite different from problem-solving although it incorporates the features in all problem-solving definitions. It was further stated that the situations must be interpreted like problems, important factors must be selected, relationships must be defined, these relationships must be interpreted mathematically, opinions must be analyzed and solution must be achieved on the situation in mathematical modeling.

4. Modeling Process

Lesh and Doerr (2003) and Blum and Niss (1991) mention the following processes as problem solving activities in mathematical modeling:

a) Understanding and simplifying the problem; understanding tables, graphs and verbal information and drawing inferences from them,

b) Manipulating the problem and developing a mathematical model; identifying the variables and the relationships among them; constructing hypotheses; evaluating contextual information, and developing models,

c) Interpreting the shared solution; making decisions, analyzing the system and proposing new solutions.

d) Verifying and showing the problem; generalizing and sharing solutions; evaluating the solution from different perspectives.

We go between the reality and mathematics while performing modeling. Modeling process begins with a complicated real-life situation. A problem representation is obtained from that situation. From here, a mathematical model is obtained by means of mathematization. The solution can be found via a mathematical study that is performed on the model. This solution is firstly interpreted, and then, its correctness is shown. If the solution or the selected process does not accord with the reality, certain stages or the entire modeling process is repeated (Doruk, 2010).

All aspects of modeling process:

- Students are given an experience regarding how mathematics can contribute in understanding, formalizing and applying the problems in different subject areas.
- Students can apply the models by defining the simple relationships in the nature, and realize the potentials and constraints of the models.
- Students can comment and discuss on the realities of the existing models.
- Students can move between the theoretical and practical aspects of modeling and problem-solving related mathematics (Blomhøj and Kjeldsen, 2006).

When the researches of mathematics educators are examined in recent years, it is observed that there is a requirement to find more efficient methods and strategies that will draw all students to meaningful mathematical learning, make them feel that mathematics is part of their lives and make them enjoy mathematics. Moreover, these methods and strategies must be able to equip the students with the rapidly advancing technology for their professional lives that they will experience throughout their existence. Furthermore, there is a necessity to make sure that they gain mathematical skills which will support them in order to find their ways efficiently in complicated situations that they will come across in their daily lives and find practical solutions to their daily problems. Modeling activities come out as a fairly efficient tool that involves the features that can meet these requirements, and they're considerably suitable to be used by mathematics instructors.

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