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A Fuzzy Approach to Reliability Analysis

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Abstract

Fuzzy probability is the foundation of the majority of currently used methods for fuzzy reliability analysis. This study uses fuzzy differential equations to characterise fuzzy reliability. Some unpredictable parameters have an impact on a system's dependability in real-world applications. By using fuzzy parameters, fuzzy reliability is a method of presenting the dependability function in an uncertain manner. Two forms of fuzzy derivatives—the Hukuhara derivative and the generalised differentiability—are used in the proposed fuzzy differential equation for dependability. The Hukuhara differentiability is shown to be insufficient for fuzzy reliability analysis. The idea of fuzzy mean time to failure (FMTTF) will then be taught utilising fuzzy integration. In comparison to the Hukuhara differentiability results for fuzzy reliability analysis, some numerical simulations are shown to demonstrate the application and validity of generalised differentiability.

Keywords: Fuzzy reliability; Fuzzy differential equation; Fuzzy derivative.

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1. Introduction

Reliability, according to He and Zhang (2016), is the probability that a network will successfully complete the tasks it has been given within a certain amount of time. Numerous authors have proposed various methods for investigating the fuzzy dependability of industrial systems. According to Chen's (1994) approach, each component of the system's dependability is represented by a triangular fuzzy number, and fuzzy number arithmetic operations are utilised to analyse the reliability of fuzzy systems. Mon and Cheng (1994) developed the functions of fuzzy numbers to be used for the fuzzy system reliability rather than employing the classical probability distribution for the components when using non-linear programming approaches and commercial software. A new approach to discover the analytical solution of VDEs with a new representation of the trapezoidal vague set (TrVS), known as JMD TrVS, was also proposed by Kumar and Lata (2012). They studied the concepts of fuzzy differential equations (FDEs) and extended vague differential equations (VDEs). There are numerous more references that have used the theory of fuzzy systems to describe uncertainty in real-world settings, including Effati and Pakdaman (2010), Effati et al. (2011), Sadoghi Yazdi et al. (2008), and Pakdaman and Effati (2016). One of the most difficult subjects for system analysts or plant workers to understand is reliability analysis, which is necessary if the system is to function more reliably over time. Unfortunately, failure is an inevitable occurrence in an industrial system, making the function of the system analyst crucial for maintaining the operation of industrial systems in order to boost their productivity and performance.

Furthermore, the fuzzy functions of series systems and parallel systems were discussed, respectively. The Bayesian reliability estimation under fuzzy environments was proposed in Wu's paper (Wu (2004)). In order to apply the Bayesian approach, the fuzzy parameters were assumed to be fuzzy random variables with fuzzy prior distributions. He transformed the original problem into a non-linear programming problem. This non-linear programming problem was then divided into four sub-problems to simplify the computation. Finally, He obtained the failure rate and reliability by solving the proposed sub-problems. Regarding the fuzzy mean time to failure (FMTTF), Liu et al. (2007) defined the MTTF of non-repairable systems with fuzzy random lifetimes.

Thus, studying the fuzzy reliability from a fuzzy differential equation perspective may be crucial from both a theoretical and practical standpoint. Based on the idea of fuzzy differential equations and fuzzy derivative, we introduce and define the fuzzy dependability function in this study. It will be demonstrated that only one of the two currently accepted definitions of fuzzy derivative produces a legitimate fuzzy reliability function, whereas the other definition produces an invalid fuzzy reliability function.The following is how the paper is set up: A few introductions are provided in the next section. The fuzzy reliability analysis based on two different types of fuzzy derivatives is introduced in Sections 3 and 4. The notion of FMTTF based on fuzzy integration is introduced in Section 5. Some numerical examples are provided in Section 6 to demonstrate the accuracy and effectiveness of the suggested strategy. Section 7 concludes by making some observations.

Preliminaries

In this section we present some necessary preliminaries from fuzzy set theory, which will be used in this paper.

Definition 2.1 (Gomes et al. (2015)) Let U be the universe of discourse. A fuzzy subset $\tilde{A}$ is determined by the following membership function:

$$\mu_{\tilde{A}} : U \to [0,1] \tag{2.1}$$

Note that in (2.1), if we replace the interval $[0,1]$ with the binary set $\{0,1\}$, then $\hat{A}$ characterizes a crisp subset of U.

Definition 2.2 (Khastan and Rodrguez-Lpez (2015)) A fuzzy set $\tilde{A}$ in the universe U, with an upper-semicontinuous membership function $\mu_{\tilde{A}}$ is said to be a fuzzy number if, $\tilde{A}$ be fuzzy convex (i.e. $\mu_{\tilde{A}}(\lambda x + (1-\lambda) y) \le \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}$ for each $x, y \in U$ and $\lambda \in [0,1]$), fuzzy normal (i.e. $\exists x \in U$ such that $\mu_{\tilde{A}}(x) = 1$) and compact support (i.e. $cl\{x \in U : \mu_{\tilde{A}}(x) > 0\}$ is compact). We denote by F(R) the set of all fuzzy numbers defined over the real set R.

Definition 2.3 (Khastan and Rodrguez-Lpez (2015)) For $0 < \alpha \le 1$ the α -cut of a fuzzy number $\hat{A}$ is defined as $[\tilde{A}]^{\alpha} = \{x \in U : \mu_{\tilde{A}}(x) \ge \alpha\}$ and for $\alpha = 0$ we define: $[\tilde{A}]^{0} = cl\{x \in U : \mu_{\tilde{A}}(x) > 0\}$, where 'cl' denotes the closure of the set. Usually, the α -cut of a fuzzy number $\tilde{A}$ is denoted by $\left[\tilde{A}\right]^{\alpha} = \left[A^{L}(\alpha), A^{U}(\alpha)\right]$ where $A^{L}(\alpha)$ and $A^{U}(\alpha)$ demonstrate the lower and upper bounds of $\tilde{A}$ respectively.

Note that $\left[A^{L}(\alpha), A^{U}(\alpha)\right]$ determines a valid α -cut of a fuzzy number, just when for each $\alpha \in [0,1]$ we have $A^{L}(\alpha) \le A^{U}(\alpha)$

Definition 2.4 Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers with α -cuts $\left[A^{L}(\alpha), A^{U}(\alpha)\right]$ and $\left[B^{L}(\alpha), B^{U}(\alpha)\right]$ respectively. Then we define:

- $\left[\tilde{A} + \tilde{B}\right]^{\alpha} = \left[A^{L}(\alpha) + B^{L}(\alpha), A^{U}(\alpha) + B^{U}(\alpha)\right],$
- $\left[\tilde{A} - \tilde{B}\right]^{\alpha} = \left[A^{L}(\alpha) - B^{U}(\alpha), A^{U}(\alpha) - B^{L}(\alpha)\right],$
- $\left[\tilde{A}.\tilde{B}\right]^{\alpha} = \left[\min_{r,s \in \{L,U\}} A^{r}(\alpha).B^{s}(\alpha), \max_{r,s \in \{L,U\}} A^{r}(\alpha).B^{s}(\alpha)\right],$
- For $\lambda \in R, \left[\lambda \tilde{A}\right]^{\alpha} = \lambda \left[\tilde{A}\right]^{\alpha}$.

Definition 2.5 (Gomes et al. (2015); Bede (2013)) The Hukuhara difference (H-difference) of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is defined as $\left[\tilde{A} -_{H} \tilde{B}\right]^{\alpha} = \left[A^{L}(\alpha) - B^{L}(\alpha), A^{U}(\alpha) - B^{U}(\alpha)\right]$.

However, the H-difference for two arbitrary fuzzy numbers doesn't exist, other differences are defined such as generalized Hukuhara difference

$$\left[\tilde{A} -_{gH} \tilde{B}\right]^{\alpha} = \left[\min\left\{A^{L}(\alpha) - B^{L}(\alpha), A^{U}(\alpha) - B^{U}(\alpha)\right\}, \max\left\{A^{L}(\alpha) - B^{L}(\alpha), A^{U}(\alpha) - B^{U}(\alpha)\right\}\right]$$

and also generalized difference.

Definition 2.6 (Gomes et al. (2015)) the distance between two fuzzy numbers $\left[\tilde{A}\right]^{\alpha} = \left[A^{L}(\alpha), A^{U}(\alpha)\right]$ and $\left[\tilde{B}\right]^{\alpha} = \left[B^{L}(\alpha), B^{U}(\alpha)\right]$ can be defined as follows:

$$d(\tilde{A}, \tilde{B}) = \sup_{0 \le \alpha \le 1} \left\{ \max\left[\left|A^{L}(\alpha) - B^{L}(\alpha)\right|, \left|A^{U}(\alpha) - B^{U}(\alpha)\right| \right] \right\} \tag{2.2}$$

Definition 2.7 (see Ahmad et al. (2013)) A mapping $\tilde{f} : (a,b) \to F(R)$ which assigns a fuzzy Number $\tilde{f}(t)$ to each $t \in (a,b)$ with α -cut $\left[\tilde{f}(t)\right]^{\alpha} = \left[f^{L}(t,\alpha), f^{U}(t,\alpha)\right]$ is called a fuzzy function (i.e. for each $t \in (a,b)$, $\left[\tilde{f}(t)\right]^{\alpha}$ denotes valid α -cut). Then f is called differentiable at $\hat{t} \in (a,b)$ if there exists an element $\tilde{f}'(\hat{t}) \in F(R)$ such that:

for all $h > 0$ sufficiently small, the H-differences $\tilde{f}(\hat{t} + h) - \tilde{f}(\hat{t}), \tilde{f}(\hat{t}) - \tilde{f}(\hat{t} - h)$ exist and:

$$\lim_{h \to 0^{+}} d\left(\frac{\tilde{f}(\hat{t} + h) - \tilde{f}(\hat{t})}{h}, \tilde{f}'(\hat{t}) \right) = \lim_{h \to 0^{+}} d\left(\frac{\tilde{f}(\hat{t}) - \tilde{f}(\hat{t} - h)}{h}, \tilde{f}'(\hat{t}) \right) = 0 \tag{2.3}$$

(II) for all $h>0$ sufficiently small, the H-differences $\tilde{f}(\hat{t})-\tilde{f}(\hat{t}+h)$ and $\tilde{f}(\hat{t}-h)-\tilde{f}(\hat{t})$ exist and:

$$\lim_{h\to 0^+} d\left(\frac{\tilde{f}(\hat{t})-\tilde{f}(\hat{t}+h)}{h},\tilde{f}'(\hat{t})\right)=\lim_{h\to 0^+} d\left(\frac{\tilde{f}(\hat{t}-h)-\tilde{f}(\hat{t})}{h},\tilde{f}'(\hat{t})\right)=0 \qquad (2.4)$$

In this situation, $\tilde{f}'(\hat{t})$ is called the type-I and type-II fuzzy derivative of $\tilde{f}$ at $\hat{t}$ respectively.

Theorem 2.1 (see Ahmad et al. (2013)) let $\tilde{f}:(a,b)\to F(R)$ and suppose that $\left[\tilde{f}(t)\right]^{\alpha}=\left[f^{L}(t,\alpha),f^{U}(t,\alpha)\right]$ for $\alpha\in[0,1]$.

(i) If $\tilde{f}$ is (I)-differentiable at all $t\in[a,b]$ then $f^{L}(t,\alpha)$ and $f^{U}(t,\alpha)$ are differentiable functions and we have $\left[\tilde{f}'(t)\right]^{\alpha}=\left[\frac{d}{dt}f^{L}(t,\alpha),\frac{d}{dt}f^{U}(t,\alpha)\right]$.

(ii) If $\tilde{f}$ is (II)-differentiable at all $t\in[a,b]$ then $f^{L}(t,\alpha)$ and $f^{U}(t,\alpha)$ are differentiable functions and we have $\left[\tilde{f}'(t)\right]^{\alpha}=\left[\frac{d}{dt}f^{U}(t,\alpha),\frac{d}{dt}f^{L}(t,\alpha)\right]$.

Definition 2.8 Suppose that $\lambda(t)$ is the hazard function and $R(t)$ is the reliability at time t.

Then, the reliability can be calculated by solving the following ordinary differential equation:

$$\frac{dR(t)}{dt}=-\lambda(t)R(t) \quad , \quad R(0)=1 \qquad (2.5)$$

Where $\lambda(t)$ is a non-negative function.

Theorem 2.2 (See Stefanini et al. (2006)): Suppose that $\tilde{f}(t)$ is a fuzzy function with α -cut $\left[\tilde{f}(t)\right]^{\alpha}=\left[f^{L}(t,\alpha),f^{U}(t,\alpha)\right]$ then:

$$\left[\int_a^b \tilde{f}(t)dt\right]^{\alpha}=\left[\int_a^b f^{L}(t,\alpha)dt,\int_a^b f^{U}(t,\alpha)dt\right] \qquad (2.6)$$

Theorem 2.2 will be used for deriving the definition of FMTTF.

2. Fuzzy reliability analysis I

In real world, reliability of a product, which determines the quality of a product during the time without failure, is not a crisp value. Indeed, the reliability of a product is affected by several unknown and known parameters. Since the reliability is calculated using hazard function and this function is not also a crisp value, thus considering the reliability as a fuzzy value is a natural way to model the imposed uncertainty. There are several approaches to define fuzzy reliability. In this section, we use (2.5) to introduce the fuzzy reliability function.

To introduce the fuzzy reliability function, based on (2.5), we must solve the following fuzzy differential equation:

$$\frac{d\tilde{R}(t)}{dt}=-\tilde{R}(t)\lambda(t) \qquad (3.7)$$

Wherein $\tilde{R}(t)$ is the fuzzy reliability function with α -cut $\left[\tilde{R}(t)\right]^{\alpha} = \left[R^L(t,\alpha), R^U(t,\alpha)\right]$.

We also consider the initial condition $\tilde{R}(0) = \tilde{1}$ where $\tilde{1}$ is fuzzy one with alpha-cut $\left[\tilde{1}\right]^{\alpha} = \left[c^L(\alpha), c^U(\alpha)\right]$.

In comparison with other existing approaches for fuzzy reliability (e.g. Gonzalez-Gonzalez et al. (2014)) which are based on fuzzy probability density function, in our proposed approach we introduce the fuzzy reliability function based on fuzzy differential equation.

To solve (3.7), if we use the first definition of fuzzy derivative (and noting that $\lambda(t)$ is positive) then:

$$\begin{cases} \dfrac{dR^L(t,\alpha)}{dt} = -R^U(t,\alpha)\lambda(t), R^L(0,\alpha) = c^L(\alpha) \\ \dfrac{dR^U(t,\alpha)}{dt} = -R^L(t,\alpha)\lambda(t), R^U(0,\alpha) = c^U(\alpha) \end{cases} \tag{3.8}$$

Equations (3.8) present a system of ordinary differential equations which can be solved analytically.

If $\lambda(t) = \lambda$ be a positive constant, then (3.8) is a time-invariant linear system. Also when λ a function of time, then (3.8) is describes a time-variant linear system. In the case of $\lambda(t) = \lambda$, system (3.8) can be presented as follows:

$$\begin{bmatrix} R^L(t,\alpha) \\ R^U(t,\alpha) \end{bmatrix}' = \begin{bmatrix} 0 & -\lambda \\ -\lambda & 0 \end{bmatrix} \begin{bmatrix} R^L(t,\alpha) \\ R^U(t,\alpha) \end{bmatrix} , \quad \begin{bmatrix} R^L(0,\alpha) \\ R^U(0,\alpha) \end{bmatrix} = \begin{bmatrix} c^L(\alpha) \\ c^U(\alpha) \end{bmatrix} \tag{3.9}$$

In this case the eigenvalues of the coefficient matrix are $-\lambda$ and λ . Thus the final solution is:

$$\begin{cases} R^L(t,\alpha) = \dfrac{1}{2}k_1 e^{\lambda t} + \dfrac{1}{2}k_2 e^{-\lambda t} \\ R^U(t,\alpha) = \dfrac{1}{2}k_2 e^{-\lambda t} - \dfrac{1}{2}k_1 e^{\lambda t} \end{cases} \tag{3.10}$$

Where $k_1 = c^L(\alpha) - c^U(\alpha)$ and $k_2 = c^L(\alpha) + c^U(\alpha)$. As it can be observed, in (3.10), when $t \to +\infty$, the values of upper and lower reliability functions tends to $+\infty$. Thus the final fuzzy reliability function is not valid. To illustrate the solution, consider the following example.

Example 3.1 Suppose that $\lambda = 10^{-1}$ and $\left[c^L(\alpha), c^U(\alpha)\right] = [0.9 + 0.1\alpha, 1]$. Then solution of (3.10) is:

$$\begin{cases} R^L(t,\alpha) = \dfrac{1}{2}k_1 e^{\lambda t} + \dfrac{1}{2}k_2 e^{-\lambda t} \\ R^U(t,\alpha) = \dfrac{1}{2}k_2 e^{-\lambda t} - \dfrac{1}{2}k_1 e^{\lambda t} \end{cases} \tag{3.11}$$

As it can be observed in Figure 1, the reliability decreases by time and the fuzzy reliability solution is a valid fuzzy function. Also the proposed fuzzy reliability function is a valid fuzzy function (since $R^L(t,\alpha) \le R^U(t,\alpha)$ for each $t \in [0,10]$). But if we extend the time horizon we observe that the values of reliability function tends to $+\infty$ (see Figure 2). Also, based on Figure 2 it can be observed that we have negative values and also values more than one for the lower and upper reliability functions which are incorrect. This also indicates that the first definition of fuzzy derivative is inconsistent for fuzzy reliability.

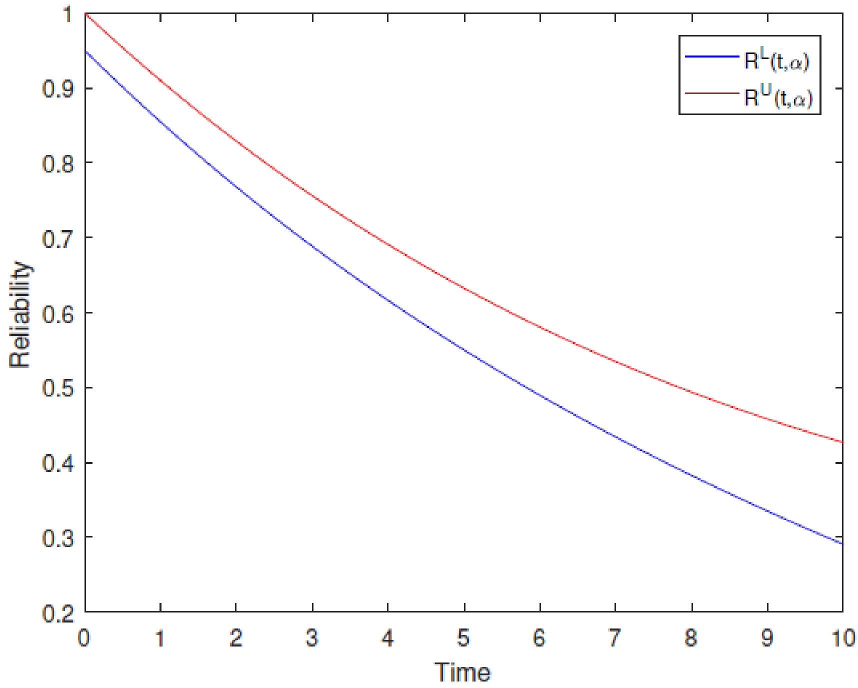

Figure 1. Fuzzy reliability in Example 3.1 in the time horizon [0; 10] for $\alpha = 0.5$

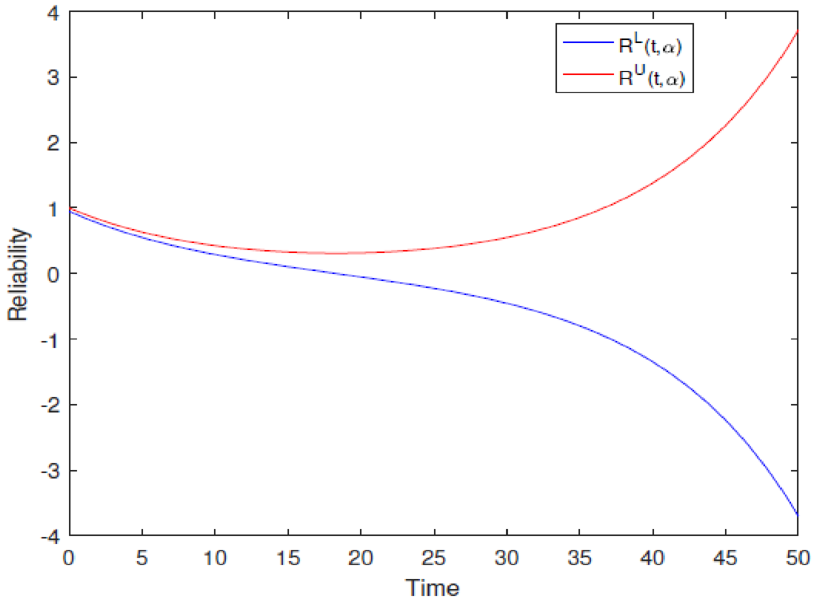

Figure 2. Fuzzy reliability in Example 3.1 in the time horizon [0; 50] for $\alpha = 0.5$

3. Fuzzy reliability analysis II

As it was discussed in previous section, the first definition of fuzzy derivative is not suitable for fuzzy reliability analysis. Now consider equation (3.7) based on the second definition of fuzzy derivative. In this sense, we have the following system of differential equations:

$$\begin{cases} \dfrac{dR^{L}(t,\alpha)}{dt} = -R^{L}(t,\alpha)\lambda(t), R^{L}(0,\alpha) = c^{L}(\alpha) \\ \dfrac{dR^{U}(t,\alpha)}{dt} = -R^{U}(t,\alpha)\lambda(t), R^{U}(0,\alpha) = c^{U}(\alpha) \end{cases} \quad (4.12)$$

Each equation in (4.12) can be solved separately. In the case of $\lambda(t) = \lambda$ the final analytical solution is:

$$\begin{cases} R^{L}(t,\alpha) = c^{L}(\alpha)e^{-\lambda t} \\ R^{U}(t,\alpha) = c^{U}(\alpha)e^{-\lambda t} \end{cases} \quad (4.13)$$

As it can be observed, in (4.13), when $t \to +\infty$the values of upper and lower reliability functions tends to zero. Since $c^{L}(\alpha) \le c^{U}(\alpha)$for each$\alpha \in [0,1]$, thus $R^{L}(t,\alpha) \le R^{U}(t,\alpha)$. To illustrate the solution, consider the following example.

Example 4.1 In Example 3.1, if we apply (4.13), then:

$$\begin{cases} R^{L}(t,\alpha) = (0.9+0.1\alpha)e^{-\lambda t} \\ R^{U}(t,\alpha) = c^{U}(\alpha)e^{-\lambda t} \end{cases} \quad (4.14)$$

The solution for time horizon [0; 10] and [0; 50] can be observed in Figures 3 and 4 respectively.

4. Fuzzy mean time to failure

In this section we try to introduce and define the fuzzy mean time to failure (FMTTF) based on definition of fuzzy integral. In crisp case, the MTTF can be calculated as follow:

$$MTTF = \int_{0}^{+\infty} R(t)dt \quad (5.15)$$

In real world applications, since the reliability is not crisp, consequently the value of MTTF could not be determined clearly. Indeed, fuzzy reliability function will result fuzzy values for MTTF.

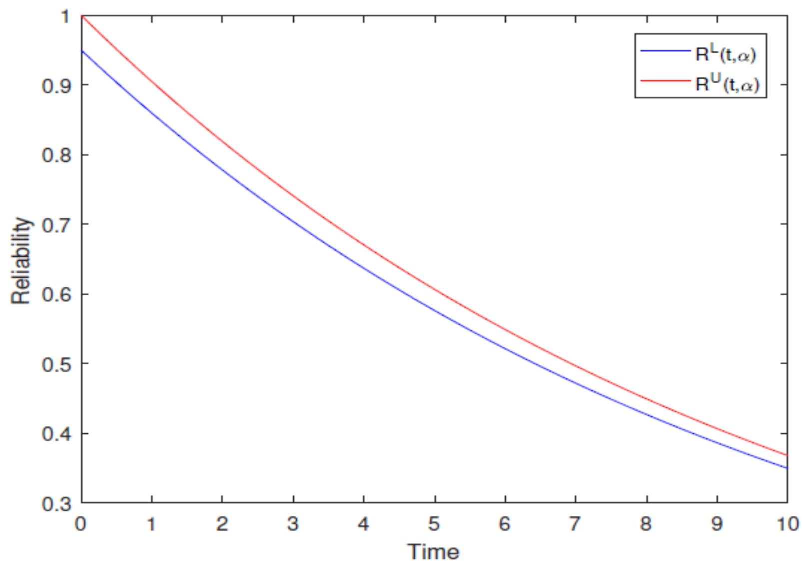

Figure 3. Fuzzy reliability in Example 4.1 in the time horizon [0; 10] for$\alpha = 0.5$

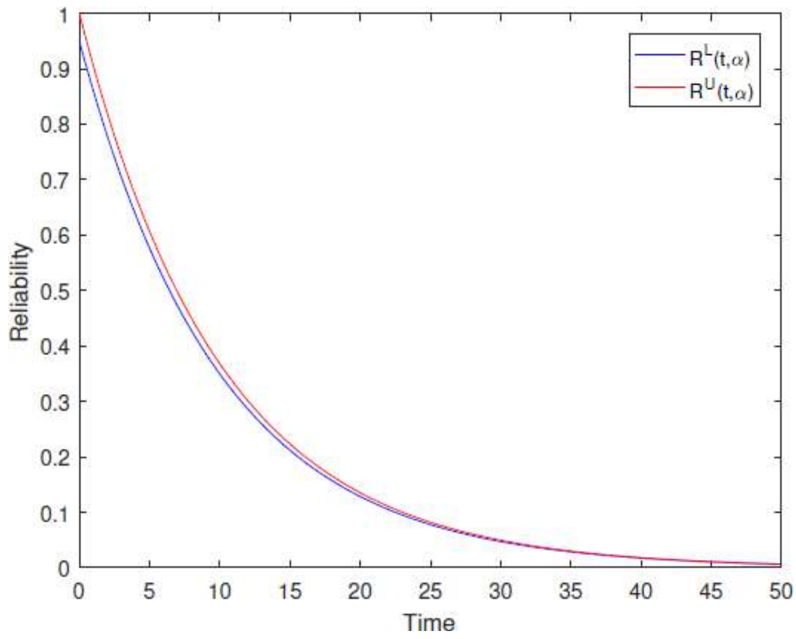

Figure 4: Fuzzy reliability in Example 4.1 in the time horizon [0; 50] for $\alpha = 0.5$

In (5.15) if we replace crisp reliability function R(t) with its fuzzy version $\tilde{R}(t)$ which now is calculated from (3.7), then we have:

$$FMTTF = \int_0^{+\infty} \tilde{R}(t)dt \tag{5.16}$$

which gives a fuzzy number. Suppose that for each $t \in [0,+\infty)$ we have $\left[\tilde{R}(t)\right]^\alpha = \left[R^L(t,\alpha), R^U(t,\alpha)\right]$. Since the upper and lower values of fuzzy reliability function are positive, thus, the α -cut of FMTTF can be calculated as follows:

$$\left[MTTF\right]^\alpha = \left[\int_0^{+\infty} R^L(t,\alpha)dt, \int_0^{+\infty} R^U(t,\alpha)dt\right] \tag{5.17}$$

Example 5.1 Suppose that we are going to calculate the FMTTF for Example 4.1. From Example 4.1 we had $R^L(t,\alpha) = (0.9+0.1\alpha)e^{-\lambda t}$ and $R^U(t,\alpha) = e^{-\lambda t}$. Thus we have:

$$\left[MTTF\right]^\alpha = \left[\int_0^{+\infty} (0.9+0.1\alpha)e^{-\lambda t}dt, \int_0^{+\infty} e^{-\lambda t}dt\right] = \left[\frac{(0.9+0.1\alpha)}{\lambda}, \frac{1}{\lambda}\right] \tag{5.18}$$

For $\lambda = 10^{-6}$ we have $\left[MTTF\right]^\alpha = 10^6\left[(0.9+0.1\alpha),1\right]$ which is plotted in Figure 5. As it can be observed in Figure (5), the values of FMTTF can be calculated for each $\alpha \in [0,1]$

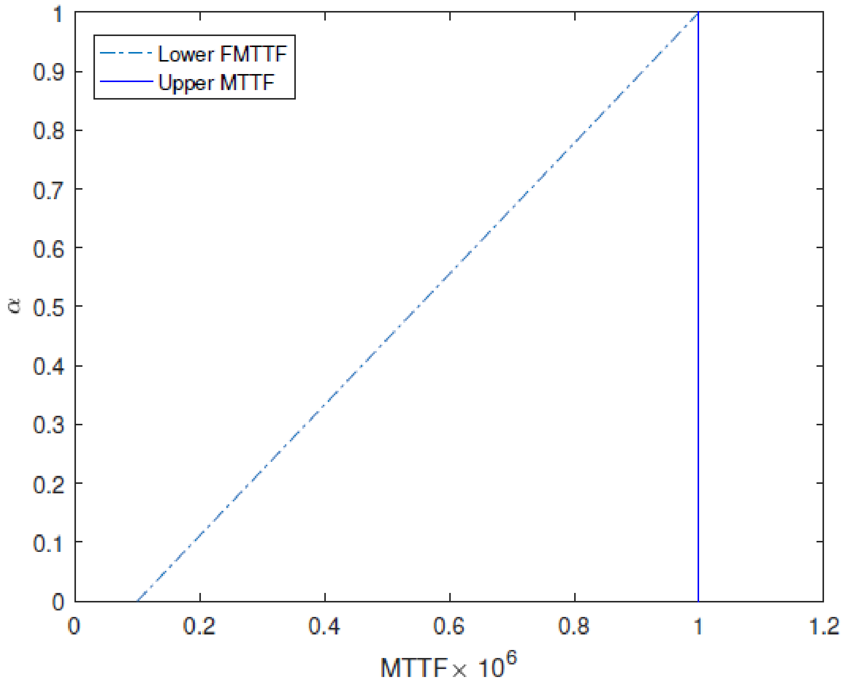

Figure 5. The value of FMTTF in 5.1

5. Illustrative examples

For the purpose of illustration, two numerical examples are provided here.

Example 6.1 In previous numerical examples, we considered a constant failure rate $\lambda(t) = \lambda$. Now, consider a special product, which has a failure factor $\lambda(t) = 0.0003t$. In this case the failure factor is a linear function of time which means that as time goes on, the failure increases with rate 0:0003. Also suppose that $\left[\tilde{R}(0)\right]^{\alpha} = [0.99 + 0.01\alpha, 1]$.We are going to calculate the reliability. The proposed fuzzy differential equation is as follows:

$$\begin{cases} \dfrac{dR^{L}(t,\alpha)}{dt} = -R^{L}(t,\alpha)\lambda(t), R^{L}(0,\alpha) = 0.99 + 0.01\alpha \\ \dfrac{dR^{U}(t,\alpha)}{dt} = -R_{U}(t,\alpha)\lambda(t), R_{L}(0,\alpha) = 1 \end{cases} \quad (6.19)$$

The solution can be observed in Figure 6. As it can be observed in Figure 6, both lower and upper bounds of fuzzy reliability function tend to zero as time tends to infinity. The different values of upper and lower reliability functions at t = 0 is due to fuzzy initial value of fuzzy reliability.

To illustrate the solution, the final fuzzy reliability solution of (6.19), is plotted in Figure 7 for time interval [0, 25]. As it can be observed in Figure 7, the final fuzzy reliability function has a triangular shape similar to the triangular initial condition. For example, when *t=25*, the fuzzy reliability $\tilde{R}(25)$ is a triangular fuzzy number which can be obtained from Figure 7. This fuzzy number is plotted in Figure 8 for different values of α .

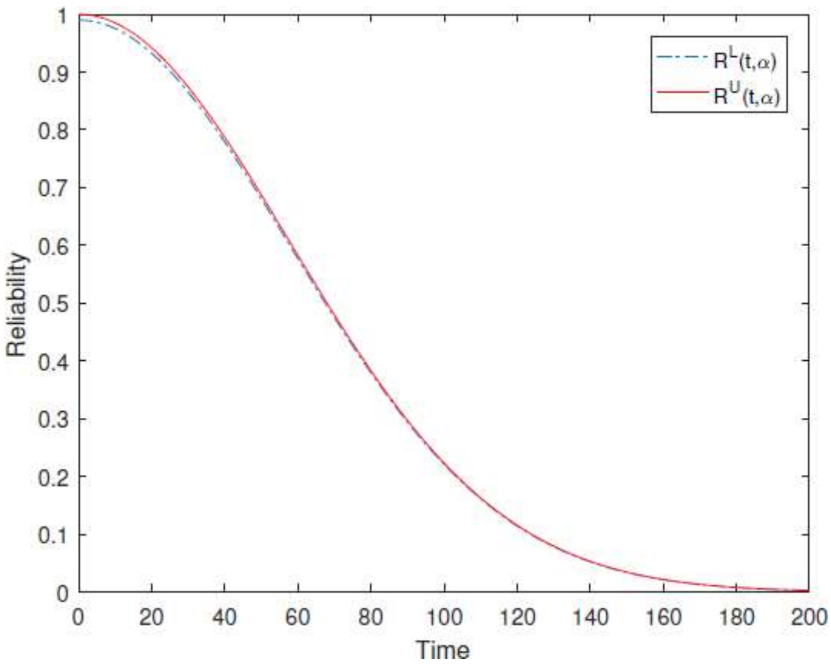

Figure 6. Fuzzy reliability in Example 6.1 in the time horizon [0; 200] for $\alpha = 0.5$

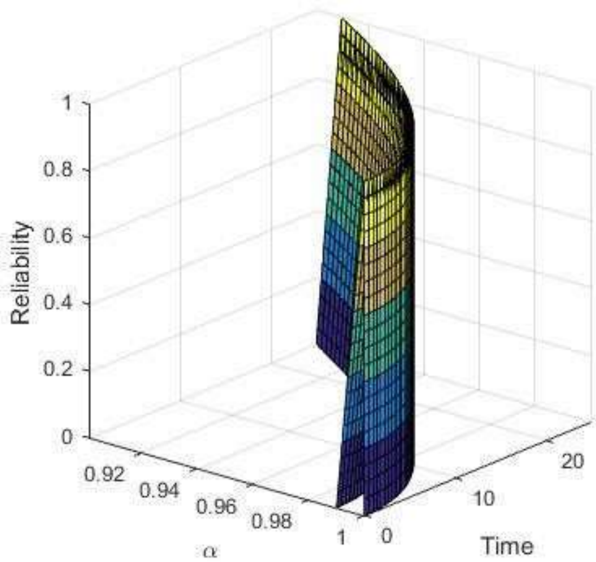

Figure 7. Final solution of Example 6.1

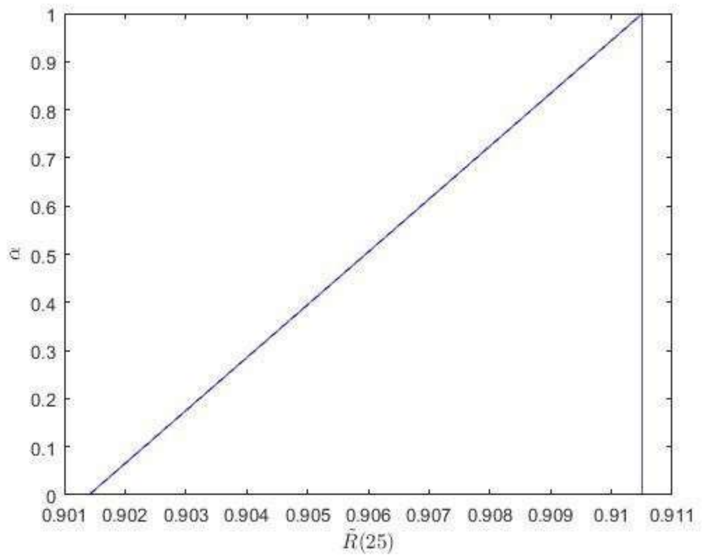

Figure 8. The fuzzy number $\tilde{R}(25)$ for Example 6.1

Example 6.2 In this numerical example, we use the proposed problem in Jamkhaneh (2011). Instead of a FDE approach, the author considered a fuzzy hazard value "about 0.7 to 0.85". Since, in this paper we use a crisp hazard function, suppose that $\lambda = 0.775$ which is the mean value of the fuzzy hazard function. Based on the second definition of fuzzy derivative, the fuzzy reliability solution for $\alpha = 0.5$ is plotted in Figure 9. Also, Figure 10 presents the fuzzy reliability $\tilde{R}(5)$. Note that $\tilde{R}(5)$, is a fuzzy number which shows the value of fuzzy reliability at time *t=5*.

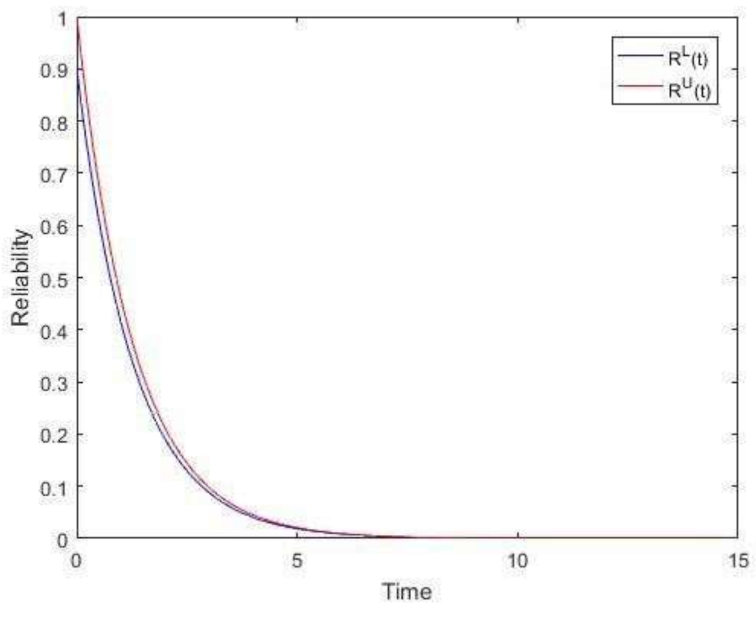

Figure 9. Fuzzy reliability in Example 6.2 in the time horizon [0; 200] for $\alpha = 0.5$

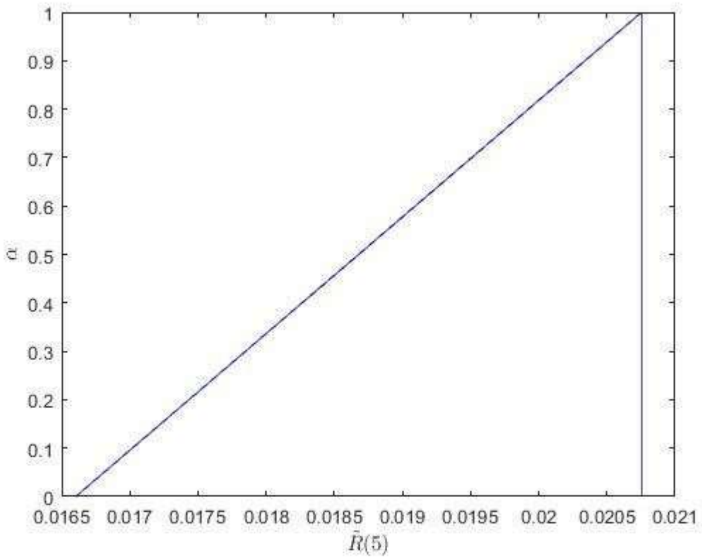

Figure 10. The fuzzy number $\tilde{R}(5)$ for Example 6.2

Note that in comparison with Jamkhaneh (2011), we used the ability of fuzzy differential equation to obtain a fuzzy reliability function and fuzzy MTTF. Also, in this paper we used two different definitions of fuzzy derivative and proved that the second one is more accurate and applicable for fuzzy reliability analysis, thus, our approach is more general.

6. Conclusions and future works

In this study, fuzzy differential equations were used to present the fuzzy reliability function. The fuzzy reliability function was interpreted and presented using two different types of fuzzy derivatives. The validity of just one of these categories for fuzzy reliability has been established. Additionally, a number of numerical simulations were presented to support and illustrate the suggestion. Fuzzy integral technology was used to introduce fuzzy mean time to failure (FMTTF). In this study, we presented the concept of fuzzy reliability by employing the concept of fuzzy derivative and fuzzy differential equation, in contrast to other existing approaches that are based on fuzzy probability distribution. Future research can use the suggested method to examine the reliability of both parallel and serial systems.

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