

Electromotive force (emf) for the confused

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Abstract

High school and undergraduate physics courses teach the electromotive force (emf), however due to the many ways this quantity can be expressed in various contexts, students are frequently perplexed. Also, while teaching pupils, the terms "potential difference" and "emf" are sometimes used interchangeably, which can lead to a number of misunderstandings. This paper aims to demonstrate the difference between the two quantities from their definitions and through the use of several examples, including the conceptual challenges when it comes to voltmeter measurements. This is expected to be of great interest to educators and students alike, as proper teaching of the topic will allow students to have a more thorough understanding of the effects of electromagnetic fields.

Keywords: emf, voltage, potential difference, electromotive force, battery, emf source, voltmeter measurements

1. Introduction

The electrical action generated by a non-electric source is known as a "electromotive force" (emf, denoted by and measured in volts) in electromagnetism [1]. Although in this situation the word "force" is inappropriate because it often refers to the contact between two bodies, the term "emf" is frequently employed for historical reasons. When Alessandro Volta created the first battery, commonly known as the "voltaic pile," in 1800, he also coined the phrase. Transducers are devices that create an emf by transforming non-electrical energy into electrical energy. The most widely known examples of emf sources are batteries, which convert chemical energy, and generators, which convert mechanical energy. Sometimes an analogy to water pressure is used to describe emf [2].

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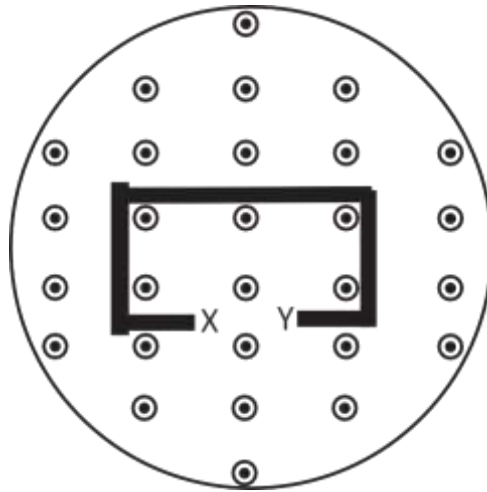


Figure 1. C-shaped conductor in a non-constant but uniform magnetic field.

The concepts of emf and how its existence can be induced by magnetism are of particular interest to high school students, being covered in many introductory electrodynamics textbooks [3–6]. However, it is a source of confusion for students due to the multiple forms that it can take [7]. In particular, the topic of induced emf may pose challenges to students, as there is a heavy focus on quantitative analysis of the phenomena, but not enough conceptual understanding on this abstract topic [8].

In high school physics, emf is often treated to be interchangeable with potential difference or voltage. This may not be a potential issue in problem-solving at the high school level, but it is a fatal fallacy. Unfortunately, this misconception has been very common, and it imposes difficulty in learning more advanced concepts of electromagnetism in higher levels of education. Emf and potential difference indeed share the same units and their values are equal in most cases. However, understanding the origins of emf may lead one to conclude that there is a great disparity between the concepts of potential difference and the concepts of emf, issues that are not addressed in many textbooks.

The thorough elaboration presented in this paper is motivated by the following question taken from a high school physics exam:

A C-shaped conductor is in a uniform magnetic field \mathbf{B} out of the page, and this field is increasing over time. What is the polarity of the induced emf in terminals X and Y? Which terminal has a higher electric potential? (figure 1)

Although the question may seem simple even for high school and undergraduate physics students, it can be misleading, especially if it is not approached from correct conceptual understanding, as elaborated in this paper.

This paper seeks to address the issues faced by physics teachers, lecturers and students in teaching and learning the concepts of emf. In the first part, the definitions of potential difference and emf will be established. In the second part, proper applications of each concept in various scenarios will be elaborated. Apart from benefiting students, this paper also proposes more

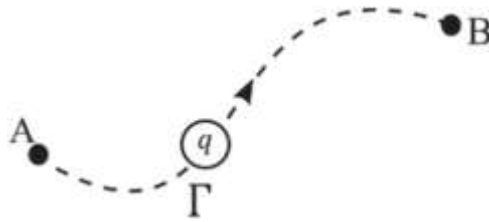


Figure 2. Shifting a test charge \$q\$ from point A to point B via path \$\Gamma\$.

2 Conceptual preliminaries

The concepts of electric potential and emf often get mixed up, and one might not be able to confidently discern one from the other. This is not surprising; both quantities carry the same unit of volts (joules-per-coulomb), and their definitions differ not in form, but in context.

Electric potential vs electromotive force

A general electrostatic setup is a collection of multiple stationary charges, and the net electric field and potential may be viewed as the superposition of the internal field, \$\mathbf{E}_{int}\$, and potential contributed by each stationary charge. We may therefore conclude that the electric field of any electrostatic setup is conservative, with an associated potential function \$\varphi\$. As the electric potential is path-independent, it is perfectly reasonable to define electric potential between points A and B, without specifying the path \$\Gamma\$ between them, i.e., \$V = -\int_{\Gamma} \mathbf{E}_{int} \cdot d\mathbf{s} = \varphi_B - \varphi_A\$. If the reference point A is fixed, the electric potential now differs from \$\varphi\$ by a constant.

In most cases, this is also the *work required per unit charge* to shift a test charge from point A to point B (figure 2), with the assumption that the shifting of the test charge is sufficiently slow and has a negligible effect on the setup. However, this physical interpretation may not hold if time dependency is allowed: moving charges create time-dependent potentials, which invalidates the slow-moving assumption of the test charge.

General electrodynamics requires consideration of the magnetic force and non-conservative induced electric fields. This means that the emf is path-dependent. If the specified path \$\Gamma\$ moves with velocity \$\mathbf{v}\$ in a region of magnetic field \$\mathbf{B}\$, we define the emf as the path integral

$$\mathcal{E} = \int_{\Gamma} (\mathbf{E}_{ext} + \mathbf{v} \times \mathbf{B}) \cdot d\mathbf{s}, \quad (1)$$

where \$\mathbf{E}_{ext}\$ represents the external electric field in the region, excluding the contribution of all the internal charge distributions within conductors. For the case of static charges in the absence of a magnetic field, equation (1) reduces to \$\mathcal{E} = \int_{\Gamma} \mathbf{E}_{ext} \cdot d\mathbf{s} = -\int_{\Gamma} \mathbf{E}_{int} \cdot d\mathbf{s} = V\$.

Consequently, the emf may be divided into two components: emf that is induced by an external electric field, we will refer to this as *induced emf*,

$$\mathcal{E}_{ind} = \int_{\Gamma} \mathbf{E}_{ext} \cdot d\mathbf{s} \quad (2)$$

and emf that is induced by the motion of the path in a magnetic field, known as *motional emf*,

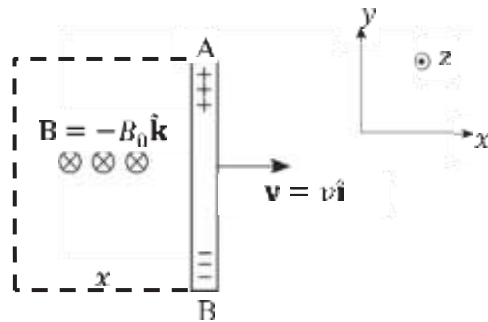


Figure 3. A conducting rod moving in a uniform external magnetic field.

$$E_{\text{mot}} = \int_{\Gamma} (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{s}. \quad (3)$$

Finally, it is also known that chemical and thermodynamic processes (e.g., redox reaction) may also generate emf via entropic means [2, 9]. The total emf is the sum of induced emf, motional emf, and thermal/chemical emf:

$$E = E_{\text{ind}} + E_{\text{mot}} + E_{\text{th/ch}}. \quad (4)$$

The above elaboration shows that emf is well defined for a path. This can also be extended to a loop, by using a closed path integral instead. At this point, it is worth re-emphasising that the emf is not a force as the name suggests, but rather an integral of force per unit charge over some length.

Faraday's law

In the early 1800s, Joseph Henry and Michael Faraday independently discovered that a time-changing magnetic flux is accompanied by an emf [10].

Consider a region of magnetic field \mathbf{B} and an arbitrary loop C . This loop may not be physical. Denote the magnetic flux through the surface S enclosed by C to be Φ_B . Faraday's law can be written as

$$E = - \frac{d\Phi_B}{dt}, \quad (5)$$

where E is the generated emf. The negative sign indicates that the direction of the induced emf is such that the induced current creates a magnetic field that opposes the change in flux, according to Lenz's law [11].

Case 1. A conducting rod AB of length l is placed parallel to the y -axis in a constant and uniform external magnetic field $\mathbf{B} = -B_0 \hat{\mathbf{k}}$. The rod moves to the right with velocity $\mathbf{v} = v \hat{\mathbf{i}}$ (figure 3). What is the emf between points A and B ?

The magnetic field is constant and uniform in the vicinity of the rod. Constructing an arbitrary loop consisting of the rod (figure 3) and applying equation (5) yield $E = - \frac{d}{dt} (B_0 l x) = -B_0 l v$. If the loop is actualized using a conducting wire, a current will start to flow.

Magnetic force, however, is not the only source of emf, as elaborated before. Experiments have shown that a current starts to flow in a closed wire that is at rest inside a time-varying

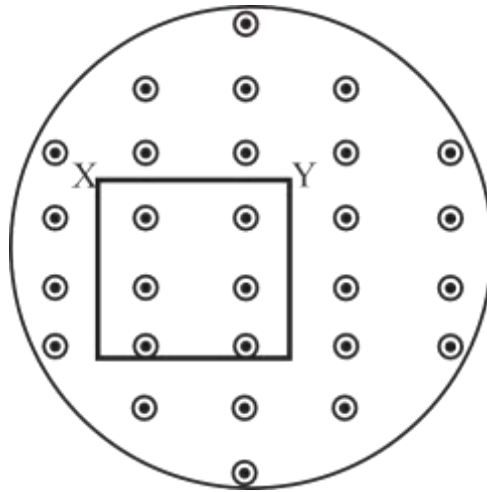


Figure 4. A conducting loop in a uniform magnetic field generated by a solenoid.

magnetic field \mathbf{B} . This is fascinating, given that the free charges in the wire were initially at rest and a magnetic field exerts forces on moving charges only! Furthermore, experiments have also shown that if the magnetic field \mathbf{B} stops varying in time, the current in the wire disappears. The only field that can put an initially stationary charge in motion and keep this charge moving is an electric field. It can then be concluded that *a time-varying magnetic field is necessarily accompanied by an induced electric field.*

From the definition of emf,

$$\mathbf{E}_{\text{ind}} = \oint_C \mathbf{E}_{\text{ind}} \cdot d\mathbf{s} = - \frac{d\Phi_B}{dt} \tag{6}$$

Even though \mathbf{E}_{ind} is a non-local phenomenon, the closed-loop integral of \mathbf{E}_{ind} depends on the local magnetic field, more precisely the magnetic flux through the loop. To illustrate this, consider the following cases.

Case 2. A square conducting loop of side a and resistance R is in a uniform magnetic field \mathbf{B} out of the page, and this field is increasing over time (figure 4). As the loop is stationary, only induced emf plays a role.

Applying equation (6) yields $\mathbf{E}_{\text{ind}} = a^2 \frac{dB}{dt}$. This induced emf drives the charges to move along the square loop in the direction specified by Lenz’s law. Hence, a clockwise induced current is produced in the loop

$$i_{\text{ind}} = \frac{\mathbf{E}_{\text{ind}}}{R} \tag{7}$$

Note that analysis of emf in a closed loop is now independent of the ‘boundary conditions of the field’. This is because the closed-loop integral may be expressed in terms of the curl of the induced electric field, which may be expressed in terms of the local change in the magnetic field.

Case 3. Refer to the very first question presented in the introduction. A C-shaped conductor is in a uniform magnetic field \mathbf{B} out of the page, and this field is increasing over time (figure 5). As the C-shaped conductor is stationary, only induced emf plays a role.

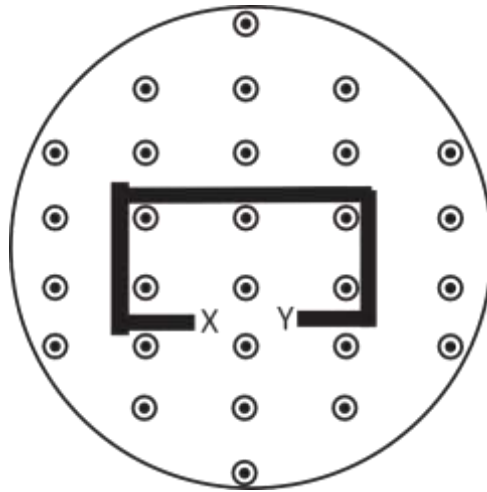


Figure 5. C-shaped conductor in the center of a uniform magnetic field generated by a solenoid.

Firstly, the uniform magnetic field may be produced by an infinitely long solenoid. Consider a cylindrical Gaussian surface that is coaxial to the solenoid. The direction of the induced electric field on the surface of the cylinder must be azimuthal, as any radial components of the electric field would imply the existence of free charges inside the Gaussian surface. Secondly, due to Gauss' law and azimuthal symmetry, we deduce that the induced electric field at any point within the solenoid only has an azimuthal component, and by Faraday's law, is directed in the clockwise direction with respect to the central axis since the magnetic field increases with time. Hence, considering a circular (imaginary) loop, smaller than and concentric to the magnetic field region (figure 6),

$$\begin{aligned} \mathbf{E}_{\text{ind}} &= \oint_C \mathbf{E}_{\text{ind}} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \\ E \cdot 2\pi r &= -\frac{d}{dt} B\pi r^2 \\ E(r) &= -\frac{r}{2} \frac{dB}{dt}. \end{aligned} \quad (8)$$

Equation (8) gives us the magnitude induced electric field at any radial position r from the central axis. It can then be integrated throughout the C-shaped conductor in figure 5. The line integral of the induced electric field from X to Y is positive, so the path Γ from X to Y along the conductor has a positive emf. The charges within the conductor will be redistributed in a state of static equilibrium, generating an electric potential exactly matching the emf. Therefore, terminal Y has a higher potential than terminal X and a net positive charge will accumulate at terminal Y.

Now, what if the C-shaped conductor is placed on a different side with respect to the central

axis of the solenoid (figure 7)? Noting that the emf of the C-shaped conductor is equal to the emf of the closed rectangular, which does not depend on its position, minus the emf along a straight

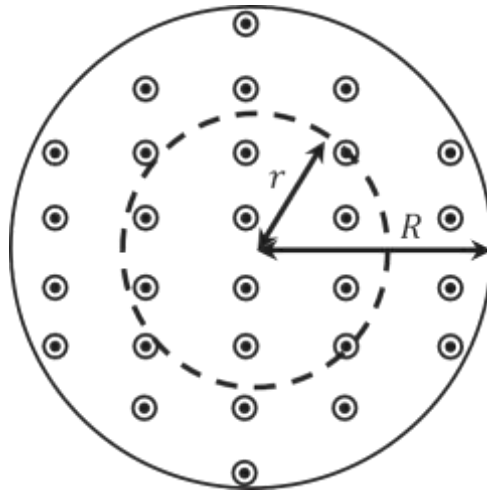


Figure 6. Imaginary circular loop, concentric to the magnetic field region.

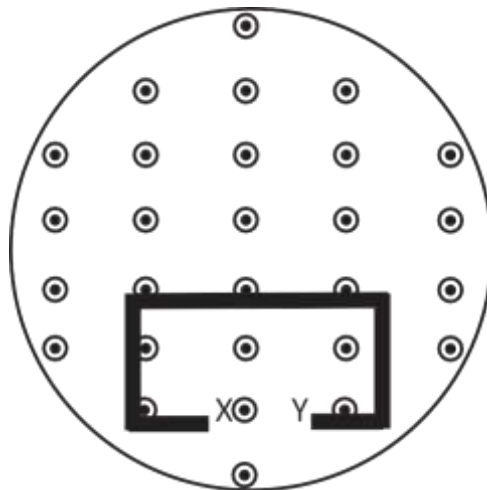


Figure 7. C-shaped conductor in a uniform magnetic field generated by a solenoid.

line from Y to X, i.e., $\int_X^Y \mathbf{E}_{\text{ind}} \cdot d\mathbf{s} = \int_C \mathbf{E}_{\text{ind}} \cdot d\mathbf{s} - \int_Y^X \mathbf{E}_{\text{ind}} \cdot d\mathbf{s}$. As the field depends linearly on r (equation (8)), offsetting the conductor far enough from the center of the field region may return a negative result! The charges within the conductor would then be redistributed in a state of static equilibrium, generating an electric potential exactly matching the emf. Therefore, terminal X now would have higher potential than terminal Y and a net positive charge would accumulate at terminal X. This shows an intriguing phenomenon: *evaluating a specific line integral may result in different signs of the emf.*

It may seem puzzling how the induced emf can depend on the position of the conductor

with respect to the region of a uniform magnetic field. The magnetic field might seem uniform

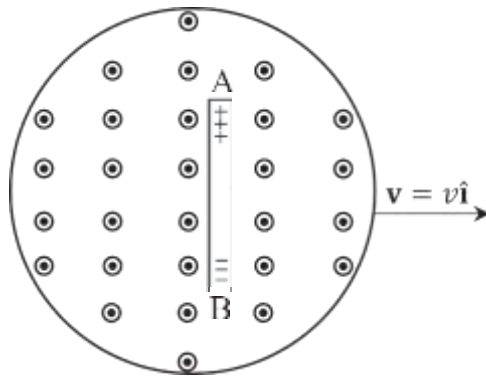


Figure 8. Conducting rod in a uniform magnetic field generated by a solenoid moving to the right.

everywhere within the solenoid, but the induced electric field is different in different regions. This is because the induced electric field due to the changing magnetic field is a non-local phenomenon, depending on the ‘boundary conditions of the field’. In most cases, we are forced to make the canonical assumption that the uniform magnetic field is generated by a solenoid aligned to the center of the shape.

3. Conceptual challenges

Moving fields

This section explores another possibility: the mechanism generating the magnetic field is moving.

EMF between two ends of conducting rod in moving uniform magnetic field. Case

4. A stationary conducting rod AB of length l is placed in a uniform magnetic field $\mathbf{B} = B_0 \hat{k}$ generated by a solenoid, which moves with velocity $\mathbf{v} = v \hat{i}$ (figure 8). What is the emf between

points A and B?

The movement of the current-carrying solenoid also generates an induced electric field. While this is not so easily derived, one may see that it must be the case: in the stationary frame of the solenoid, the rod moves to the left with velocity v and any charge q within the rod experiences force

$$\mathbf{F} = q \quad -v \hat{i} \times B_0 \hat{k} = qvB_0 \hat{j}.$$

Einstein’s first postulate in special relativity requires this to also be the case in the stationary frame of the rod. Therefore, there must be an induced electric field $\mathbf{E}_{\text{ind}} = vB_0 \hat{j}$ in the original frame, which generates the induced emf

$$\mathbf{E}_{\text{ind}} = vB_0 l \tag{9}$$

from point B to point A. At equilibrium, there will be a potential difference of the same magnitude with point A having higher potential than point B.

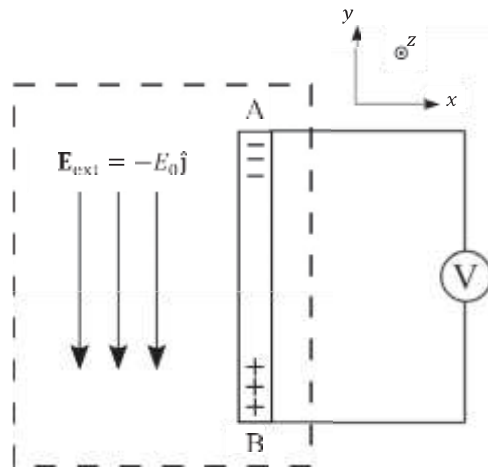


Figure 9. Conducting rod in a uniform electric field with voltmeter outside the electric field region.

Co-existence of potential difference and EMF

Potential difference and emf are both present in many cases, such as in cases 1, 3 and 4 and common DC circuits. For batteries in a closed circuit, the environment within the batteries facilitates a chemical emf to be generated across the terminals of the batteries. Meanwhile, the environment within the wires and resistors facilitates potential difference due to charge distribution and current.

Now, what is the ‘potential difference’ between point X and Y in case 2 (figure 4)? A crude application of Ohm’s law yields either $i_{ind}R/4$ or $-3i_{ind}R/4$, depending on which physical path we take. This is where the concept breaks down. How can we resolve this conflict? Note that in this case there will not be any stationary charges accumulating at any point along the loop. This is a case where emf does not co-exist with potential difference. However, this also triggers the next question: *what will a voltmeter measure when connected between points X and Y?* The answer will be further elaborated in the next section.

Voltmeter measurements

Voltmeters work by drawing a small current through the circuit and obtaining the voltage that is proportional to the current. For standard DC circuits, the emf and conservative electric fields are contained within the circuit elements themselves, so the voltmeter can be considered independent of the circuit. However, the position of the voltmeter circuit can become important in the presence of external fields.

Consider a simple example of a conducting rod AB of length l placed parallel to the y-axis in a uniform external electric field $\mathbf{E}_{ext} = E_0 \mathbf{j}$ (figure 9). After achieving the electrostatic equilibrium condition, positive charges would accumulate at point B and negative charges at point A, and the potential difference (and emf) between points A and B would be

$$\int_B$$

$$V_B - V_A = - \int_A \mathbf{E}_{\text{int}} \cdot d\mathbf{s} = E_0 l. \quad (10)$$

If the voltmeter is placed within the same electric field, we see that

$$\mathbf{E}_{\text{ext}} \cdot d\mathbf{s} = 0$$

around the circuit, thus the emf is zero and there would be no current through the voltmeter. This means that the voltmeter will display a zero reading. This result may seem counterintuitive, as the two ends of the rod are of different polarity.

To see why this is the case, we may break the circuit into two separate parts—the rod and the voltmeter component. Let Γ_1 denote the path from A to B along the rod, and Γ_2 denote the path from B to A along with the voltmeter. We have the emf around the circuit to be

$$\mathcal{E} = \int_{\Gamma_1} \mathbf{E}_{\text{ext}} \cdot d\mathbf{s} + \int_{\Gamma_2} \mathbf{E}_{\text{ext}} \cdot d\mathbf{s}.$$

This quantity is also what the voltmeter reads—if the voltmeter has internal resistance R , then the current through the circuit is $\frac{\mathcal{E}}{R}$, and the voltmeter will display a reading of \mathcal{E} . To measure the potential difference across the rod (which is incidentally equal to the emf across its two ends), we need to ensure that the second component on the right-hand side is zero. One configuration which satisfies this is where the voltmeter is outside the external electric field, and the connecting wires run perpendicular to the field, as demonstrated in figure 9.

What about the rod in case 1? Here, there is no external electric field, so only a motional emf plays a role. Let Γ be the path from A to B through the conductor. This path moves along with the rod with velocity $\mathbf{v} = v \hat{\mathbf{i}}$. Therefore, the motional emf between points A and B is obtained by invoking equation (3), $\mathcal{E}_{\text{mot}} = - B_0 v$. The negative sign appears because the magnetic force per unit charge $\mathbf{v} \times \mathbf{B}$, which is upward, opposes the direction of the path (from A to B), which is downward. If the voltmeter is moving at the same speed as the rod, there would be no current through the voltmeter. The logic for this is similar to the above-mentioned example. The current through the circuit is thus zero, and the voltmeter will show a zero reading. To measure the emf across the rod (which is also equal to the potential difference), the voltmeter circuit must be stationary with respect to the field. This is achievable by allowing the rod to slide on conducting rails to which we attach the stationary voltmeter (figure 10).

For case 3, if we connect the voltmeter to the two ends of the loop, this creates a complete circuit (albeit with high resistance), and the voltmeter will use the small current produced to yield a reading. Hence, the reading of the voltmeter will be $\int \mathbf{E}_{\text{ext}} \cdot d\mathbf{s} = - \frac{d\Phi_B}{dt}$. Here, we see phenomena that defy common sense: (1) changing the position of the voltmeter changes the magnetic flux passing through the closed circuit, and hence, the reading of the voltmeter; (2) flipping the position of the voltmeter with respect to the C-shaped structure changes the polarity of the voltmeter reading (figure 11).

If we do not want the voltmeter circuit to affect the measurement of the induced emf, we must make sure that the electric field lines are perpendicular to the voltmeter circuit. If the uniform magnetic field is generated by a solenoid, this can be done by taking the voltmeter out of the sides of the solenoid with the wires in the radial direction (figure 12).

We now address the issue of ‘potential difference’ between X and Y in case 2. Since the electric field is induced by a changing magnetic field, this means that it is non-conservative, and we cannot just rely on the usual scalar potential function anymore (unless the concept of vector potential is introduced). This can indeed be seen by integrating the electric field around one

loop of the cycle and seeing that the result is non-zero, which is not the case in a conservative field with a potential function. Hence, one cannot define a 'potential difference' between two

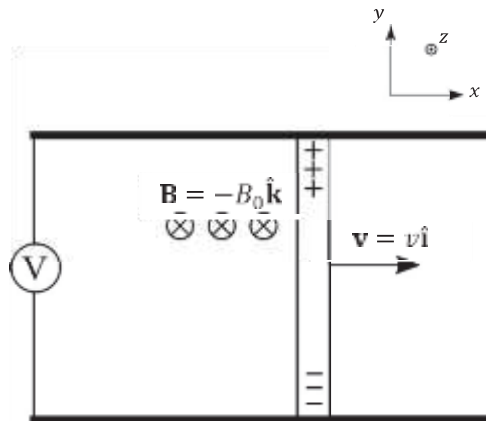


Figure 10. Conducting rod sliding along two parallel rails in a uniform magnetic field, with stationary voltmeter.

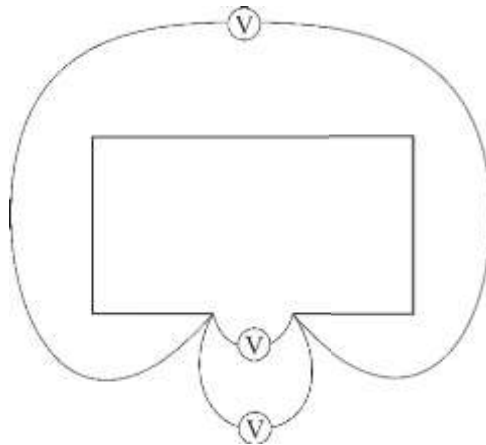


Figure 11. Voltmeter at different positions with respect to the C-shaped conductor.

points in the loop. This brings us to the question of what the voltmeter measures when its two ends are connected to X and Y.

As shown in figure 13, two identical voltmeters are attached to points X and Y, one connected above, and one connected below. Suppose they register readings of V_1 and V_2 , respectively. We now solve for the readings in terms of the resistances of the wire loop, R , and the identical voltmeters, r . Suppose that the top voltmeter loop encloses a magnetic flux of Φ_1 , the square loop encloses a magnetic flux of Φ_2 , and the bottom voltmeter loop encloses a magnetic flux of Φ_3 . We denote the time rate of change of flux Φ_i as $\dot{\Phi}_i$, where $i = 1, 2, 3$, and the currents through the wire segments as described in figure 13.

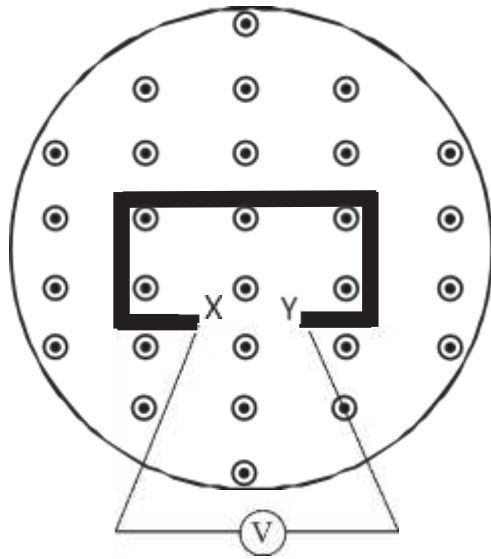


Figure 12. C-shaped wire in solenoid with axis in the z -direction and increasing current I .

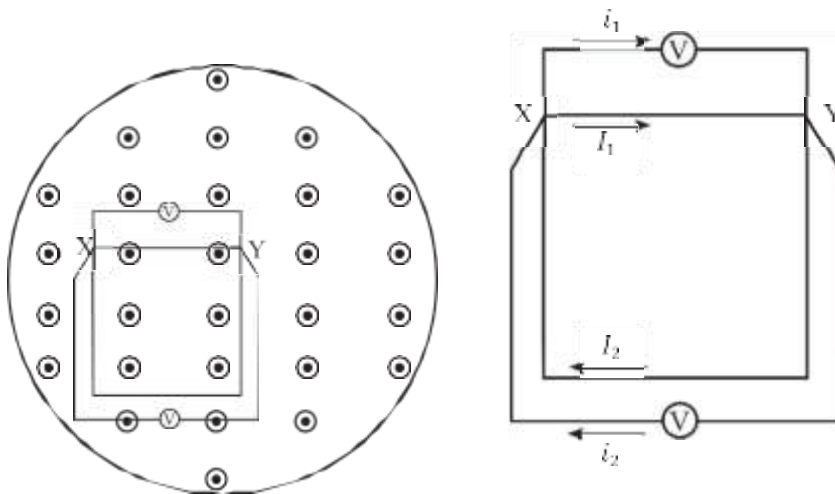


Figure 13. Two voltmeters attached in different positions to points X and Y.

We thus write down the following four equations:

$$i_1 r - I_1 \frac{R}{4} = \dot{\Phi}_1$$

$$I_1 \frac{R}{4} + I_2 \frac{3R}{4} = \Phi_2$$

$$i_2 r - I_2 \frac{3R}{4} = \dot{\Phi}_3$$

$$i_2 + I_2 = i_1 + I_1.$$

Solving the equations, we obtain

$$i_1 = \frac{(3R + 16r)\dot{\Phi}_1 + (3R + 4r)\dot{\Phi}_2 + 3R\dot{\Phi}_3}{2r(3R + 8r)}$$

$$i_2 = \frac{3R\dot{\Phi}_1 + (3R + 12r)\dot{\Phi}_2 + (3R + 16r)\dot{\Phi}_3}{2r(3R + 8r)}$$

$$I_1 = \frac{-6R\dot{\Phi}_1 + (6R + 8r)\dot{\Phi}_2 + 6R\dot{\Phi}_3}{R(3R + 8r)}$$

$$I_2 = \frac{2R\dot{\Phi}_1 + (2R + 8r)\dot{\Phi}_2 - 2R\dot{\Phi}_3}{R(3R + 8r)}$$

Applying the approximation $r \ll R$,

$$i_1 \approx \frac{1}{r} \dot{\Phi}_1 + \frac{1}{4} \dot{\Phi}_2$$

$$i_2 \approx \frac{1}{r} \left(\frac{3}{4} \dot{\Phi}_2 + \dot{\Phi}_3 \right)$$

$$I_1 \approx I_2 \approx \frac{1}{R} \dot{\Phi}_2.$$

Let us call the induced current in the wire loop, which is approximately $\frac{1}{R} \dot{\Phi}_2$, as i_{ind} . This translates to a measurement of

$$V_1 = i_1 r = \dot{\Phi}_1 + \frac{1}{4} \dot{\Phi}_2$$

$$V_1 = \dot{\Phi}_1 + \frac{1}{4} i_{\text{ind}} R \tag{11}$$

on the top voltmeter and

$$V_2 = -i_2 r = -\frac{3}{4} \dot{\Phi}_2 - \dot{\Phi}_3$$

$$V_2 = -\frac{3}{4} i_{\text{ind}} R - \dot{\Phi}_3 \tag{12}$$

on the bottom voltmeter. If the magnetic field only exists in the region bounded by the wire loop,

then $\Phi_1 = \Phi_3 = 0$, and $V_1 = {}^1i_{\text{ind}}R$, $V_2 = -{}^3i_{\text{ind}}R$ —a fascinating reality that shows that the

voltmeter displays different measurements depending on its position with respect to the loop! This counterintuitive fact has been well known and experimentally proven [12].

For case 4, we apply similar logic to case 1 to conclude that the voltmeter must be moving at the same velocity as the solenoid, for otherwise (say, the voltmeter is stationary in the lab frame), if we shift into the frame of the solenoid, we obtain the same situation as in case 1, where the voltmeter will give a zero reading.

4. Discussion and conclusion

Only for conservative electric fields does the potential difference have a clear definition. In example, the electric fields produced by static charge distributions within a conductor are commonly specified for the potential difference between two sites of the conductor. Emf, on the other hand, is path-dependent and accounts for any nearby electromagnetic fields. One might only compare the two in the case of static charge equilibrium. By carefully defining the two quantities, one may realize that there are subtleties between the two, and the examples in this paper illustrate that although they share the same physical units, they are indeed two separate quantities with distinct contexts, and both may be present in the same scenario. In several cases, an external magnetic field contributes to the emf between two points, and since the emf depends on the boundary conditions of the field, it may be necessary to specify where the magnetic field is located, as the location of the field also affects the electric field in space, and in turn, the components within the field as well.

It has also been discussed what a voltmeter measures. The placement of the voltmeter component is crucial since an outside electromagnetic field could have an impact on it. Despite the fact that the voltmeter frequently displays the potential difference between two places, it's important to remember that voltmeters only display values depending on the tiny current that they draw. This current can originate from both the emf produced by non-conservative electric fields and the potential difference brought on by the conservative electric field (from stationary charges).

The concept of emf is a puzzling topic in physics, and many seemingly simple examples challenge students' conceptual understanding of the topic. With proper teaching of the concept of emf, students would have a deeper understanding of the topic and the physical effects of electromagnetic fields.

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