International Journal of Engineering, Management, Humanities and Social Sciences Paradigms (Volume 31, Special Issue of January 2019)
ISSN (Online): 2347-601X and Website: www.ijemhs.com

The Analysis of Bridge Rectifier Circuits using *Mathematica*

[1*] **Dr. Srinibash Swain**, [2] **Mr. Pradosh Ranjan Parida,**
[1*] *Professor, Dept. Of Electrical Engineering, NIT BBSR,*
2Asst. Professor DEPT. of Electrical Engineering, NIT BBSR,
[1*] srinibashswain@thenalanda.com, , pradoshranjan@thenalanda.com

Abstract

Bridge rectifiers' steady-state and transient analyses are covered in this essay. It begins by offering a useful illustration of the diodes. The performance of the diode in both the forward and reverse orientations is described by a single equation. It implicitly considers the polarity of the voltage. The proposed method is applied to the conventional uncontrolled four-valve full-wave rectifier bridge configuration without losing its generality. Shown are the supply voltage waveform, the effect of the load resistance, and the eventual existence of a smoothing shunt-capacitor. The approach can handle cases involving non-sinusoidal sources and possible non-linear elements in the rectifier circuit. The results of a Mathematica code include the currents and voltages of all circuit elements. The suggested procedure is validated by comparing the solution of two case studies with the corresponding results available in the literature.

Keywords: *Analytical models, bridge, diodes, Mathematica, power electronics, rectifier circuits, single-phase, stead-state, transient analysis, valve stresses*

INTRODUCTION

Power electronic loads are being used more and more, which has caused severe technical and financial issues with how power networks should be designed and run [1–11]. Although they haven't had much of an effect on the harmonic voltage and current levels in these networks, because of the anticipated increasing penetration levels, the situation is projected to get worse.

The analysis presented in [4] addresses the un-controlled single- and three-phase rectifiers and their compatibility with the corresponding IEC standards.

Reference [5] describes the line side behavior of the single-phase uncontrolled rectifiers as affected by the short-circuit ratio at the valve side of the rectifier. The influence of both the finite DC link capacitance and the AC ohmic resistance is also considered and assessed.

A set of graphics could be generated which lead to a user-friendly procedure for the estimation of the line current. The obtained results were validated through the comparison with those calculated using the PSpice.

Paper [6] proposes a method for studying a combination of single- and three-phase non-linear loads. It can reduce the current harmonics significantly, as validated by a number of measurements in several applications of three-phase adjustable-speed drives (ASD).

A passive wave shaping method for single-phase diode rectifiers is presented in reference [7]. It is shown that its application can lead to higher input power factor, lower rectifier current stresses, and reduced component ratings.

The study [8] investigates the simulation of controlled rectifier circuits using the MATLAB7.0 Simulink power system simulation model. It addresses the circuit's MATLAB simulation model diagram and the resulting waveforms. In order to validate the approach, these waveforms and the corresponding theoretical results are compared. Several studies have been also performed in order to predict the harmonics impact of ASD and heat pumps on residential power distribution networks.

Reference [9] describes the effect of the supply voltage harmonics on the response of single-phase capacitor-filtered diode bridge rectifiers. A model for calculating the current harmonics of these loads, when energized by non-sinusoidal supply voltages, is presented. It was found that the voltage crest factor is a much better indicator for the total harmonic current distortion (THDI) than the total voltage distortion (THDV) and that the resulting current harmonics produce voltage harmonics with a partial self-compensating effect on the current harmonics.

Paper [10] presents experimental results pertinent to the performance of the single-phase rectifiers based on a normalized parameters' approach. The analysis considers voltage distortion levels in actual distribution systems of approximately 2–3%. A database of the rectifier harmonic currents is also proposed. It allows the fundamental and harmonic currents of any single-phase rectifier fed by a general sinusoidal supply voltage to be calculated. Three laboratory tests are developed to verify the applicability of the database.

This paper is a contribution to this topic and explores, among others, the possibility of adopting the software *Mathematica* in the simulation of rectifier circuits. The investigation will have the following features:

1. The presentation of an efficient diode model and its incorporation in the equivalent circuits and analysis of full-wave rectifier bridges.
2. The governing system of algebraic-differential equations is derived. It will take into account the time wave form of the source voltage as well as its internal resistance and inductance. The suggested technique can handle pure sinusoidal or harmonic-polluted voltage sources. Moreover, the presence of shunt smoothing capacitors across the load terminals will be also considered.

3. The suggested technique can handle nonlinear circuit elements such as load resistances and iron-cored components.

METHOD OF ANALYSIS

It is assumed that the diode has a small resistance Ohms in the forward direction (i.e.if its current $i \geq 0$) if its current and a large resistance in the reverse direction (($i \leq 0$).). Its voltage-current characteristics can therefore be expressed by the following equations:

$$If\ i \geq 0: v = i\ R_f \quad (1)$$

$$If\ i \leq 0: v = i\ R_r \quad (2)$$

These two relations can be expressed by the following single compact equation:

$v = i.R_f.\mathrm{u}[i] + i.\mathrm{R}_r.\mathrm{u}[-i]$ (3)

Or,

$$v = i.(R_f.\mathrm{u}[i] + \mathrm{R}_r.\mathrm{u}[-i]) = i.R_{eq}[i] \quad (4)$$

The term $R_e[i]$ is the diode's "equivalent resistance" as a function of the current's magnitude and sign (or polarity).

The term u[x] denotes the unit step function which is equal to 0 for $x \leq 0$ and equal to +1 otherwise.

Equation (3) is depicted values $R_f = 1\mu\Omega$, $R_r = 500\ k\Omega$, respectively in Figureure.1 for the resistance values, respectively

Figure 2 shows the circuit of a full-wave rectifier bridge connected to the voltage source *e(t)*. The source has the internal resistance and inductance. They includes the values describing the connection lead. The bridge is loaded by a resistance. The element *C* represents the eventually shunt-connected smoothing capacitor.

The following simultaneous equations can be obtained through the application of Kirchhoff's voltage and current laws:

$$v_1 - v_2 = [i_1] \quad (5)$$

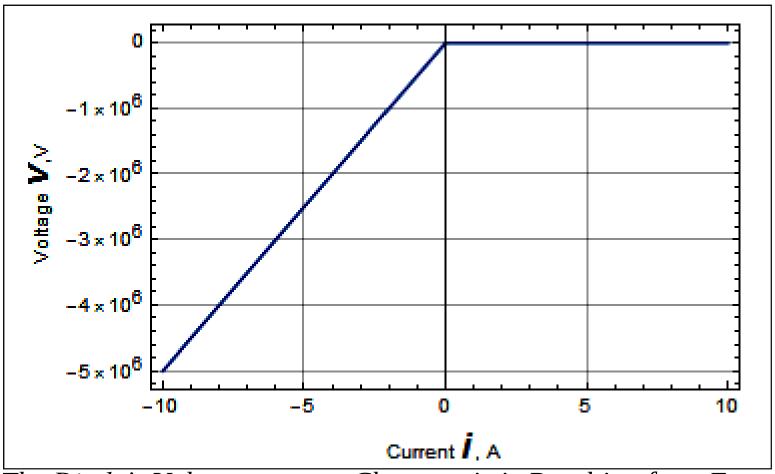


Fig. 1: *The Diode's Voltage–current Characteristic Resulting from Equation (3).*

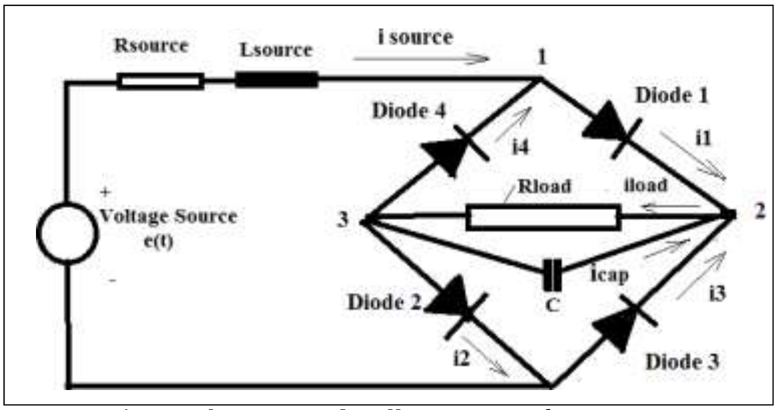


Fig. 2: *The Assumed Full-wave Rectifier Circuit.*

$$v_3 - v_1 = [i_4] \quad (6)$$

$$-v_2 = [i_3] \quad (7)$$

$$v_3 = [i_2] \quad (8)$$

$$v_1 = e - L_{source} \cdot \left(\frac{di_{source}}{dt}\right) \quad (9)$$

$$i_{source} = i_1 - i_4 \quad (10)$$

$$i_{sourc} = i_2 - i_3 \quad (11)$$

$$i_{load} = \frac{(v_3 - v_2)}{R_{load}} \quad (12)$$

$$i_{capacitor} = C.\frac{(v_3 - v_2)}{dt} \quad (13)$$

$$i_1 - i_{load} + i_{capacitor} + i_3 = 0 \quad (14)$$

A linear load will be represented by a constant value of R_{load} in equation (12), otherwise it should be expressed in terms of the load current i_{load}.

A linear load will be represented by a constant value of in equation (12), otherwise it should be expressed in terms of the load current.

The above system of 10 simultaneous algebraic-differential equations can be solved utilizing the program command *NDSolve* of the software *Mathematica* 12, as shown in Figure 3.and reference [12]. The program yields the analytical solution in terms of 10 interpolating functions for the unknown currents and voltages over a specified time range In this case the solution will be the energization transients and the steady-state response from zero initial conditions, as indicated in the box below. For more details, reference [12] should be consulted. If the steady-state response is required, the solution should be found for the time range where.

```
sol=NDSolve[{isource[t] ==i1[t] -i4[t] ,
  isource[t] ==i2[t] -i3[t] ,
  v1[t] ==e[t]-indsource* D[isource[t],t] -rsource*isource[t],
  iload[t] ==(v2[t]-v3[t])/rload,
  v1[t]-v2[t]==v[i1[t]],
  v3[t]-v1[t]==v[i4[t]],
  v3[t] ==v[i2[t]],
  i1[t]-iload[t]+icap[t]+i3[t]==0,
  -v2 [t]==v[i3[t]],
  icap[t]==cap*D[(v3[t]-v2[t]),t],
  i1[0]==0,i2[0]==0,i3[0]==0,i4[0]==0,v1[0]==0,v2[0]==0,v3[0]==
0,iload[0]==0, isource[0]==0,icap[0]==0 },{i1 ,i2 ,i3 ,i4 ,v1 ,v2 ,v3
,iload ,isource,icap },{t,0,tend},AccuracyGoal->3,PrecisionGoal->
3,MaxSteps->Infinity]
```

Fig. 3: *A Part of the Mathematica Code.*

SAMPLE RESULTS

The following results refer to a case study with the data given below:

= 0.00001 Ω
= 500000 Ω
20 Ω
= 0.001 H
= 0.001 Ω
=
=0.5 s
= V
C=
Frequency = 60-Hz
sin (377t)[7]

The Steady-State Response

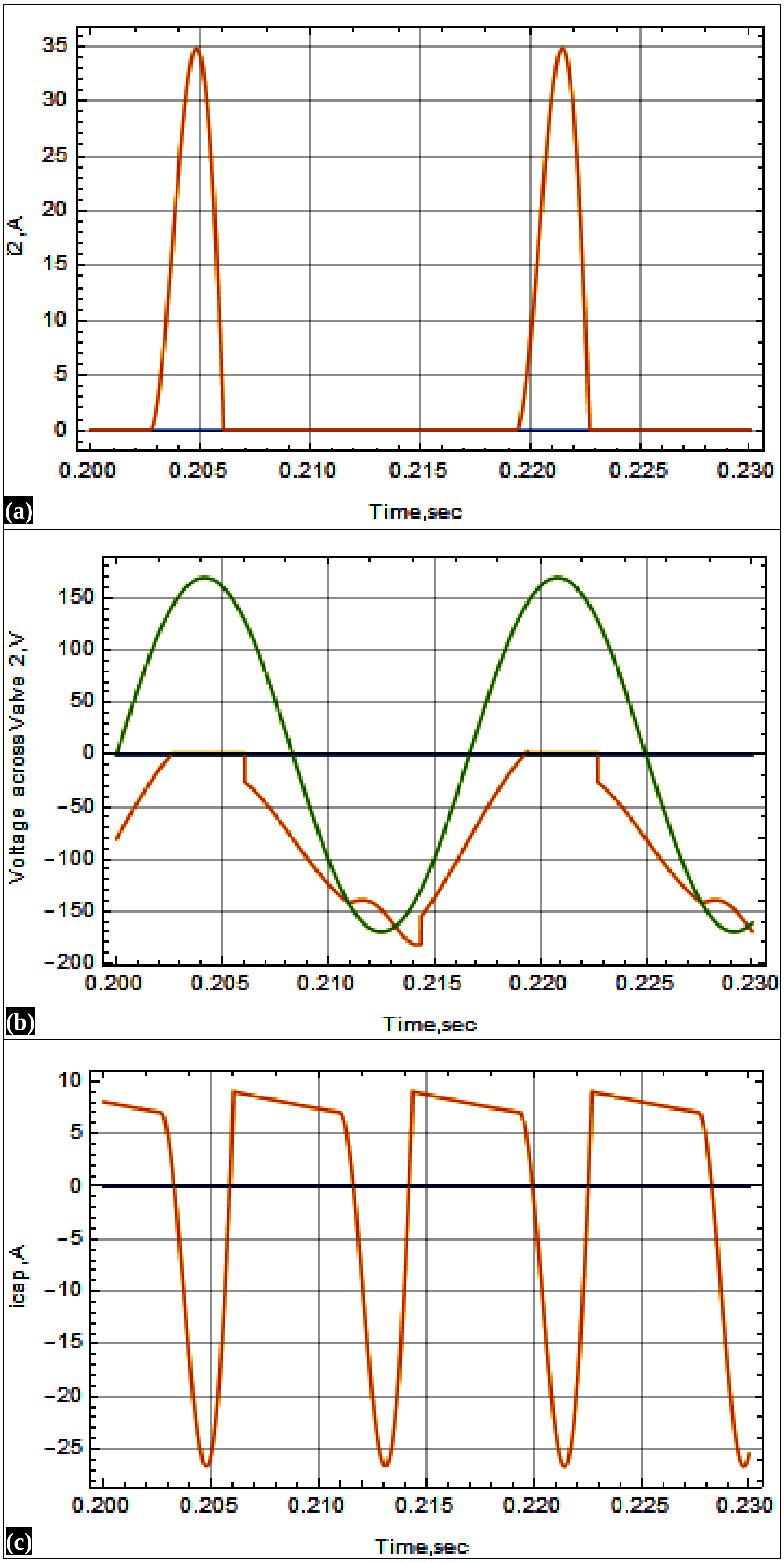
The plots of Figure 4(a-d) illustrate the steady-state response of the current flowing in diode 2, trace (a), the voltage across it, trace (b), together with the source voltage , the capacitor current , trace (c), the source current (curve d) and the load current (curve e), respectively. The last plot of Figure 4(f) (curve f) depicts the source current (multiplied by 3), the source voltage *e(t)* and the load voltage. The instantaneous value of the current through diode 2 can reach 35A. During the diode conduction, the voltage is almost zero. In the reverse direction, the voltage assumes relatively large negative values. It can momentarily reach -180V. The current and voltage conditions of the two valves 1 and 2 are almost identical. The same is valid regarding the two other diodes 3 and 4 but with a time shift of 8.333ms, which is half the voltage periodicity.

The capacitor current oscillates between -27A and +8A with sharp peaks in the negative range. The strongly distorted source current varies between -35A and +35A with the fundamental frequency 60-Hz. The rectified load current (e) exhibits a positive peak value of 35A. The source current (multiplied by 3), the pure sinusoidal 60-Hz source voltage *e(t)* together with the load voltage is shown in the last plot of Figure 4(f). The smoothing effect of the shunt capacitor can be clearly recognized.

The Transient Response

The plots in Figure 5(a-d) depict the transient response of the voltages and currents during the first 100 ms following the circuit energization. From plot of Figure 5(a), the peak value of the current flowing through diode 2 decreases gradually with time from the first peak of approximately 110A to the final steady-state value 35A.

The transient capacitor current, plot of Figure 5(c), exhibits an inrush phenomenon. The magnitude of its negative peaks decreases from about 102A immediately after switching to the steady-state value 27A. The positive peaks of the load current, plot of Figure 5(d), decrease gradually from an initial value of approximately 105A to the final value 35A.


I2,A
35
30
25
20
15
10
5
0
0.200
0.205
0.210
0.215
0.220
0.225
0.230
Time,sec
(a)
Voltage across Valve 2,V
150
100
50
0
−50
−100
−150
−200
0.200
0.205
0.210
0.215
0.220
0.225
0.230
Time,sec
(b)
icap,A
10
5
0
−5
−10
−15
−20
−25
0.200
0.205
0.210
0.215
0.220
0.225
0.230
Time,sec
(c)

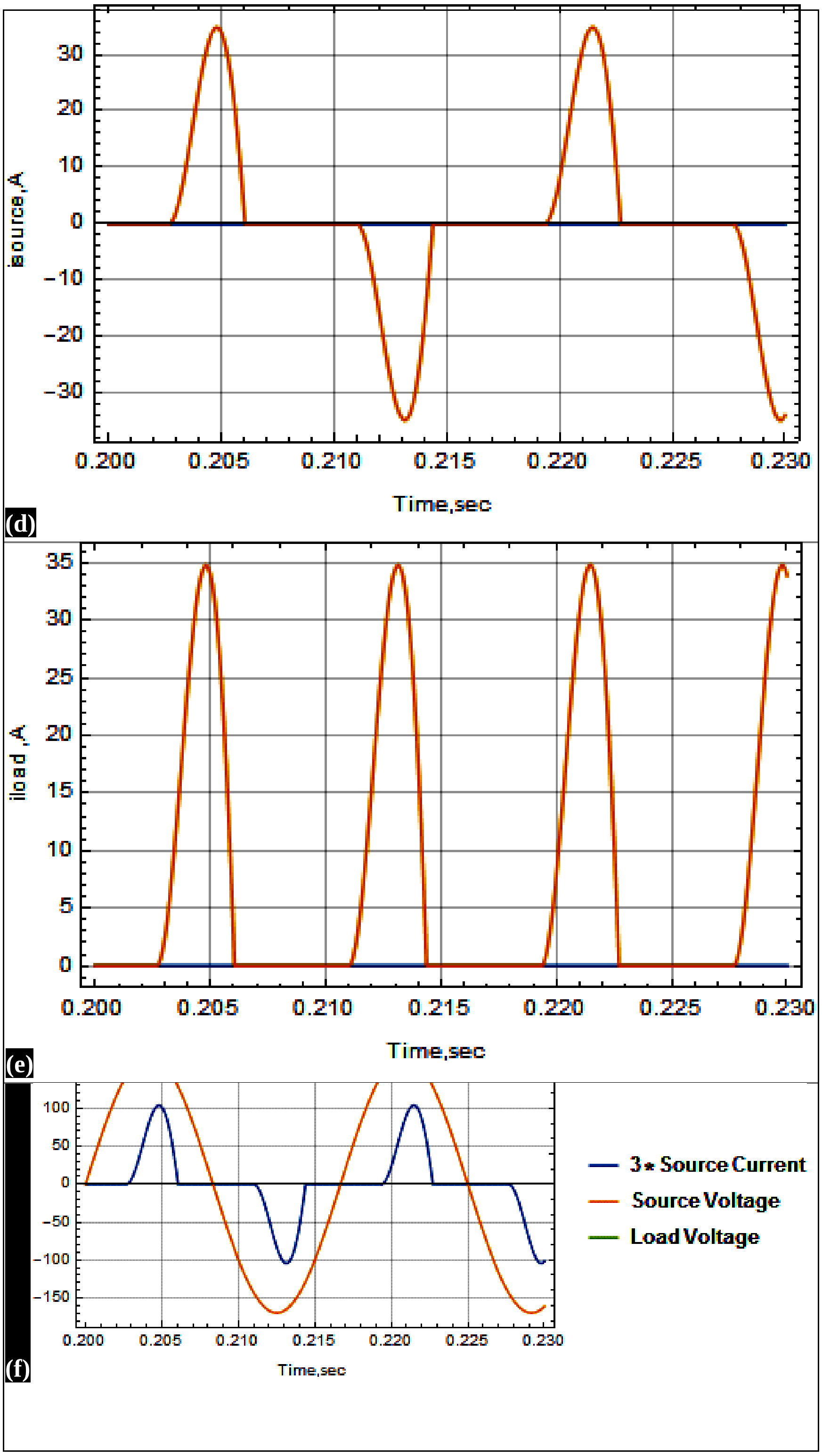
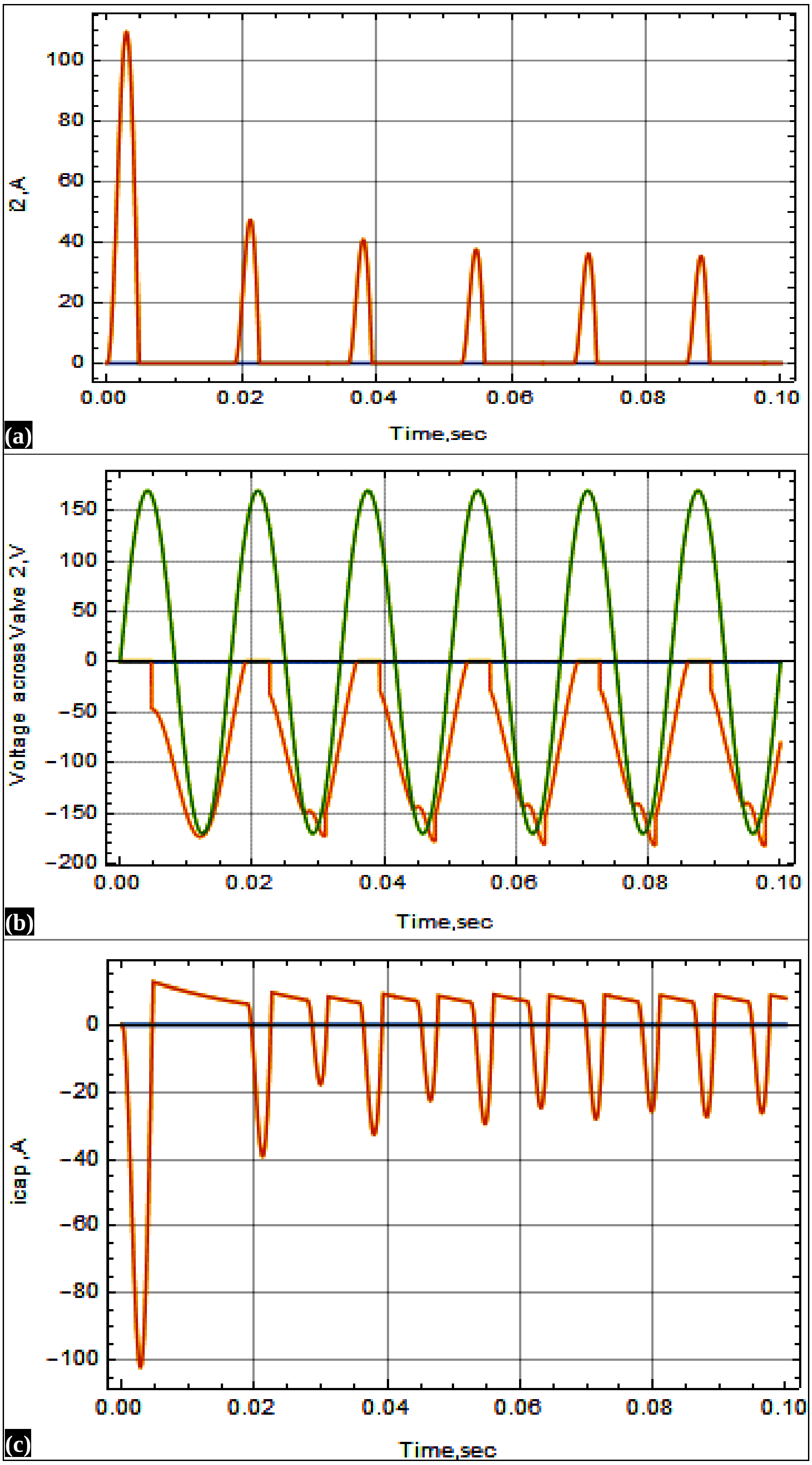


Fig. 4: *The Steady-state Response during the Time Range (200 ms).*

The positive and negative peaks of the source current decrease with time from the first positive peak of about 325A to their final steady-state values (approximately 100A) after about 40ms, as depicted in the last plot of Figure 5(f). As the plot also indicates, they coincide in time with the peaks of the source voltage. The filtering effect of the capacitor can be observed in the trace of the load voltage. The peaks in this curve decrease gradually from its first value 260V to the final peaks of about 180V. Its average value is approximately 155V, which is 8.66%less than the peak value of the source voltage (169.7V).



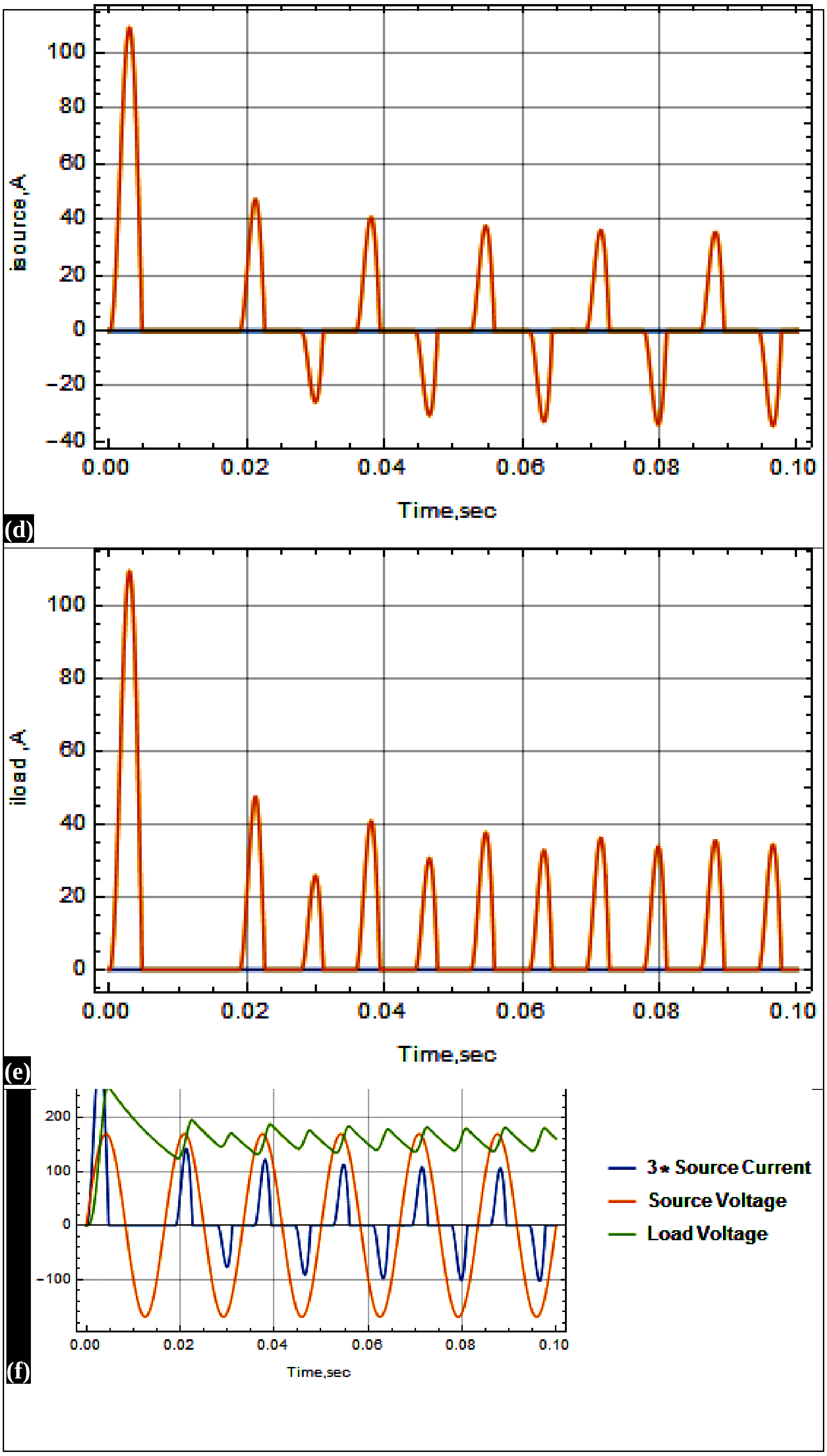


***Fig. 5:** The Transient Response during the Time Range (0 ms).*

Effect of the Smoothing Capacitor

The transients with a 25% larger smoothing capacitor (i.e. C=1250μF) can be recognized if the last plot of Figure 5 is compared with the corresponding one in Figure 6. The waveform of the source current is significantly affected in terms of the waveform and magnitude. For instance, the first peak increases from 325A to about 390A. The peak and average values of the load voltage are generally higher in comparison with Figure 5 describing the original case.

Effect of the Supply Voltage Waveform

Figure 7 illustrates the effect of the presence of a 40V third harmonic component in the source voltage, which is one third of the 60-Hz fundamental component, as described by the red curve. The source current (blue curve is highly polluted if compared with Figure 5. Its first peak increased from 325A to 430A. Moreover, the ripple content in the load voltage (in the steady-state) is smaller.

Effect of Load Nonlinearity

This section demonstrates the capability of the suggested approach to handle diode circuits including non-linear loads. Figure 9 illustrates the transient response of the rectifier bridge when supplying a non-linear resistive load having the following voltage-current characteristic, which is illustrated in Figure 8:

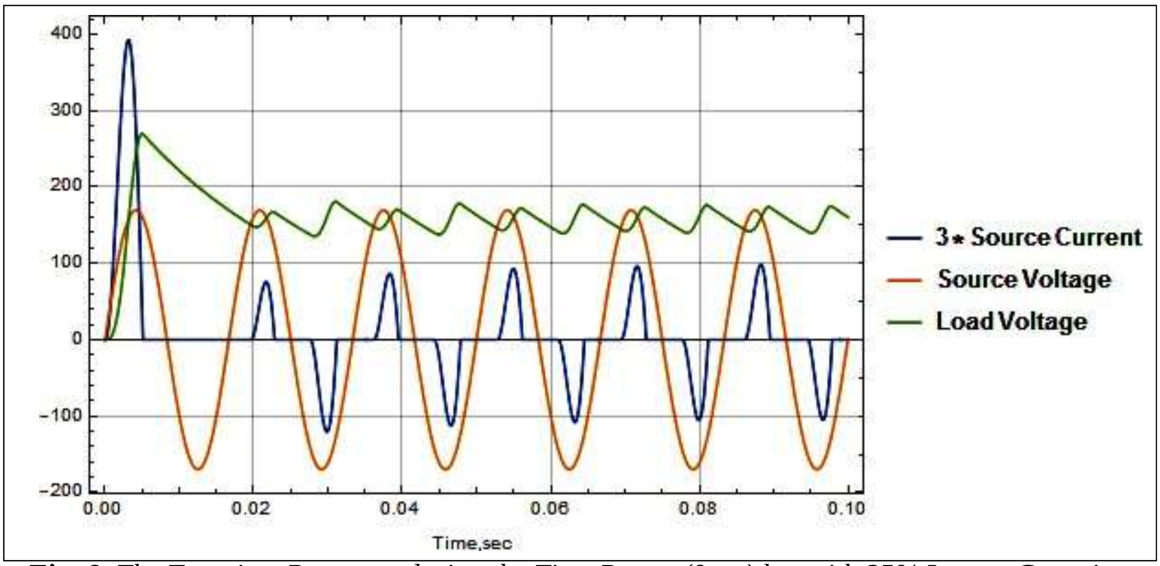


Fig. 6: *The Transient Response during the Time Range (0 ms) but with 25% Larger Capacitor (C=1250µF).*

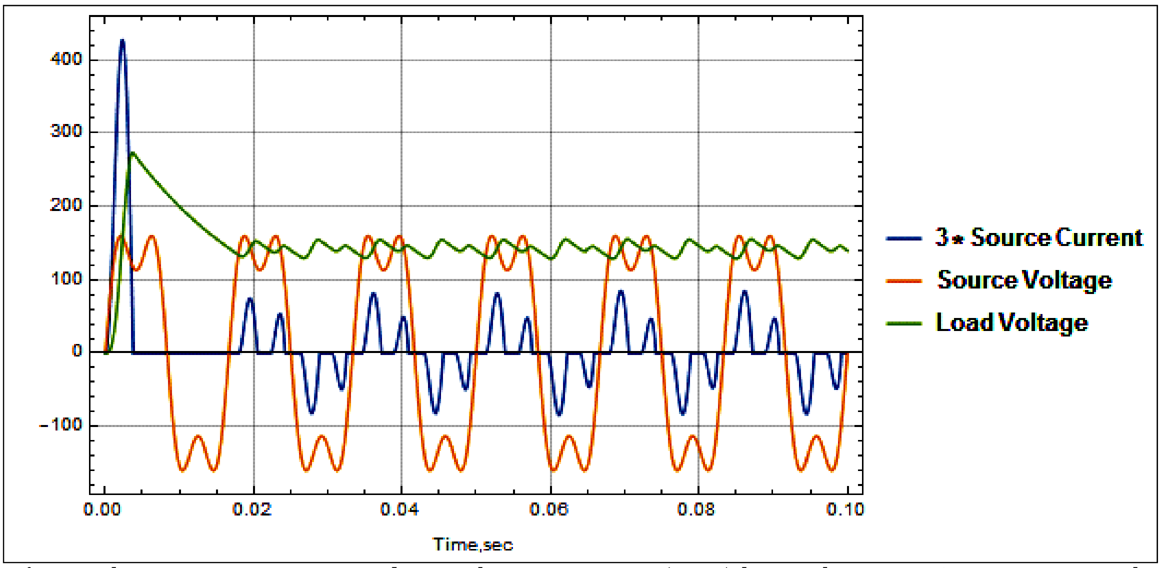


Fig. 7: *The Transient Response during the Time Range (0 ms) but with 33.333% Harmonics in the Source Voltage (C=1000µF).*

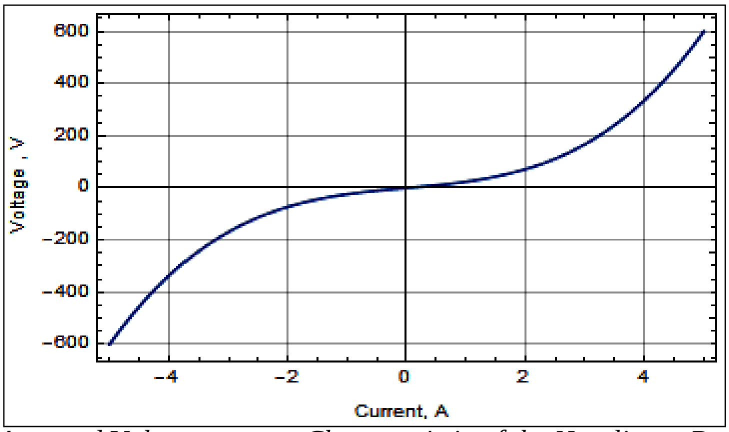


Fig. 8: *The Assumed Voltage-current Characteristic of the Non-linear Resistive Load.*

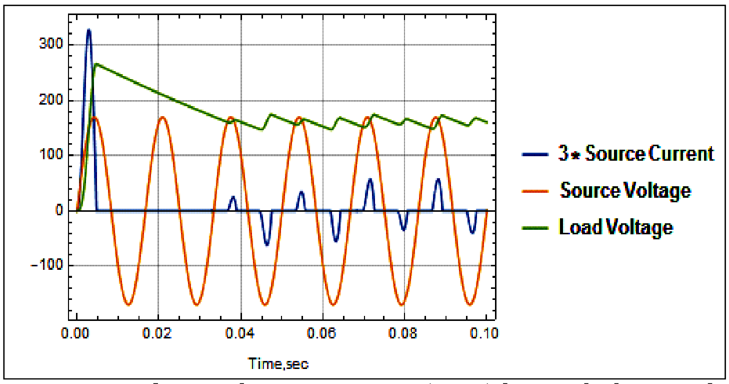


Fig. 9: *The Transient Response during the Time Range (0 ms) but with the Non-linear Resistive Load.*

This implies that the load resistance is 20Ω for very small currents and increases more than proportional with the current *i*.

The source current is seen to be always smaller than that of the 20Ω linear case depicted in Figure 5. Furthermore, the load voltage has a reduced ripple content.

VALIDATION OF THE APPROACH

The objective of this section is to validate both the *Mathematica* code as well as the introduced concept of the diode's "equivalent resistance" as defined by Equation (4).

Test 1

This test will be based on the Solved Example 5–2 on page 97 of reference [11]. It deals with a 60-Hz, full-wave bridge rectifier with V, a source inductance=1mH, a source resistance=1mΩ, *C*=1000μF capacitor and a 20Ω linear load resistance. The upper plot of Figureure.10 reproduces the results of the MATLAB solution in [11]. The chart depicts the sinusoidal source voltage and the steady-state response of both the source current and the load voltage. The lower plot shows the output obtained from the *Mathematica* code. The full agreement of both solutions can be clearly recognized.

Test 2

This test deals with a single-phase, 3kW, 240V ASD heat pump. The ASD has a smoothing capacitor of *C*=4200μF and a total source impedance of 8% of phase angle 45 degs. (based on 240V, 5kVA). The source voltage is assumed to contain a 10% third harmonic component (Flattened, F, or Peaking P). The upper chart in Figure 11 depicts the source voltage and current in both cases according to reference [9], while the lower plot shows the *Mathematica* solution for the (Flattened Case F). Again here a complete agreement between the two solutions is clear.

Voltage and current waveforms using 10% peaking (P) and flattened (F) 3rd harmonic voltages.

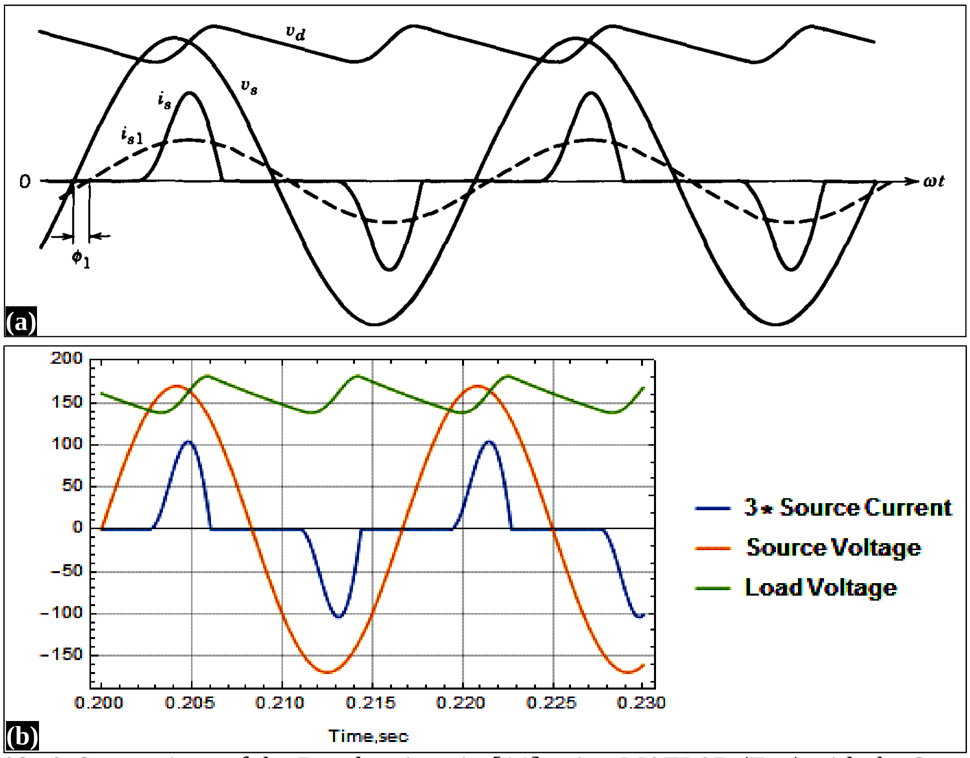


Fig. 10: *A Comparison of the Results given in [11] using MATLAB (Top) with the Suggested Mathematica Solution (Bottom) for the Same Case Study.*

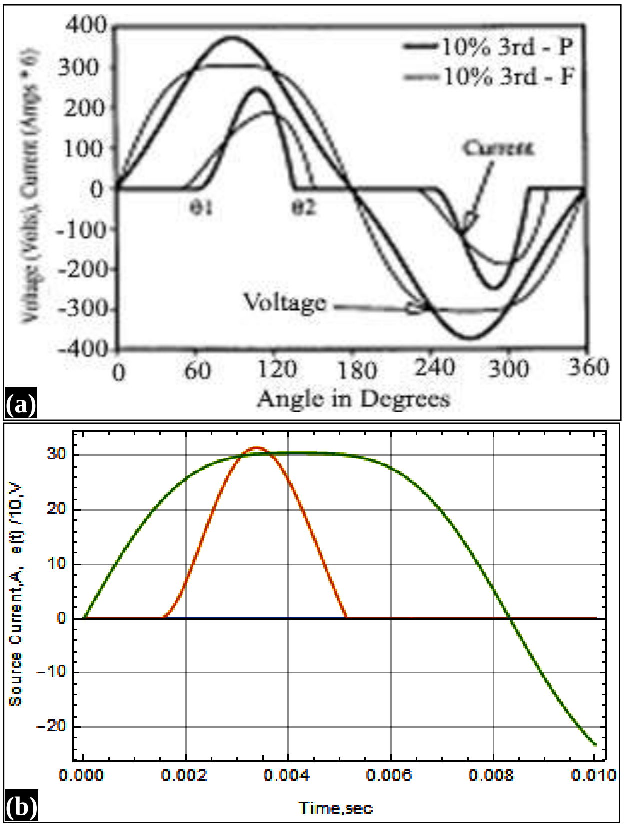


Fig. 11: *A comparison of the Results Given in [9] (Top) with the Suggested Mathematica Solution (Bottom) for the Same Case Study.*

CONCLUSIONS

It is noted and emphasised how important power electronics devices and loads are, as well as the necessity of their precise modelling. Many of the currently in use simulation approaches are included in the study as well. For the purpose of modelling diode circuits, the term "effective resistance" is introduced. It contains details on the polarity of the applied voltage, the forward and reverse resistances of the diode. Several case studies utilising a Mathematica software successfully implement this suggested representation.A computer code simulating a single-phase full-wave rectifier bridge circuitis presented. It can determine both the transient and the steady-state responses of the rectifier circuits. Non-linear loads, harmonic polluted source voltages as well as the presence of smoothing shunt capacitors can be dealt with. Two tests have been conducted in order to validate the suggested approach. Theyshow complete agreement of the obtained results with those available elsewhere in the literature and produced by other solutiontechniques.

REFERENCES

1. P.C. Sen. Power Electronics. Tata McGraw–Hill Publishing Company Limited. 1995
2. M. H. Rashid. Power Electronics, circuits, Devices and applications. Prentice – Hall , Second Edition, 1994
3. D. Yan, Alexis Kwasinski. Study of a single-phase full-wave uncontrolled rectifier with a constant power load. 2018 IEEE International TelecommunicationsEnergy Conference (INTELEC).
4. J.G. Mayordomo, A. Hernandez, R. Asensi, L.F. Beites, and M. Izzedine. A unified theory of uncontrolled rectifiers, discharge lamps and arc furnaces. PART I. An analytical approach for normalized harmonic emission calculations. Proceedings of the Eighth IEEE International Conference on Harmonics and Quality of Power (ICHQP), Athens, Greece, 1998, pp. 740–748
5. S. Herraiz, L. Sainz, J. Pedra. Behavior of single-phase full-wave rectifier. Eur.

Trans. Electr. Power 13 (3) (2003) 185–192.
6. S. Hansen, P. Nielsen, F. Blaabjerg. Harmonic cancellation by mixingnonlinear single-phase and three-phaseloads. IEEE Trans. Ind. Appl. 36 (1) (2000) 152–159.
7. A.R. Prasad, P.D. Ziogas, and S. Manias.A novel passive waves haping method for single-phase diode rectifiers". IEEE Transactions on Industrial Electronics, 37(6), (1990), pp.521–530.
8. Han-hong Tan1, Xiang Zhao. A Simulation Research on Single Phase Bridge Full Control Resistive Load Rectifying Circuit Based on MATLAB. 4th International Conference on Advanced Materials and Information Technology Processing (AMITP 2016), pp. 294–299, published by Atlantis Press
9. A. Mansoor, W.M. Grady, R.S. Thallam, M.T. Doyle, S.D. Krein, M.J. Samotyj. Effect of supply voltage harmonics on the input current of single-phase diode bridge rectifier loads. IEEE Trans. Power Delivery 10 (3) (1995)1416–1422.
10. Luis Sainz, Joaqu´ın Pedra, Juan Jose Mesas. Single-phase full-wave rectifier study with experimental measurements. Electric Power Systems Research 77 (2007) 339–351.
11. N. Mohan, T. M. Undeland, W.P. Robbins. Power Electronics, Converters, Applications and Design. Book, Second Edition, John Wiley, Chaper 5, pp. 95–100.
12. Wolfram Mathematica Tutorial Collection.Advanced Numerical DifferentialEquation Solving. in *Mathematica, Book*,Wolfram Research, 2008, available at:http://www.wolfram.com/learningcenter/tutorialcollection/Adv ancedNumericalDifferentialEquationSolvingInMathematica/